Verification Analysis of Spread Footing

Program: Spread Footing
File: Demo_vm_en_02.gpa

In this verification manual you will find a hand-made verification analysis of bearing capacity of a spread footing for drained conditions in a permanent design situation. Results of the hand-made calculations are compared with the results from the GEO5 – Spread Footing program.

Terms of Reference:

In Figure 1, an example of a centric spread footing is shown. The spread footing has a width $h = 1.80\ m$, a length $l = 2.20\ m$ and a height $t = 0.40\ m$. The footing bottom is in a depth $d = 1.20\ m$. The earth body is formed of silty sand (SM) and its terrain is adjusted in a $\beta = 7°$ inclination. The properties of soil (effective values) are shown in Table 1. In Figure 2, a load acting on the spread footing is shown and its values are in Table 2. The spread footing is covered with soil of unit weight $\gamma_z = 20.00\ kN/m^3$ after the realization. The spread footing is made from reinforced concrete with unit weight $\gamma_c = 23.00 kN/m^3$. The dimensions of the column on the spread footing are $c_x = 0.40\ m$ and $c_y = 0.40\ m$. The bearing capacity is calculated using the standard approach. The verification methodology is done according to EC 1997 with design approach 2 – reduction of actions and resistances. In this example the self-weight in favour $(\gamma_G = 1.00)$ and disfavour $(\gamma_G = 1.35)$ is calculated.

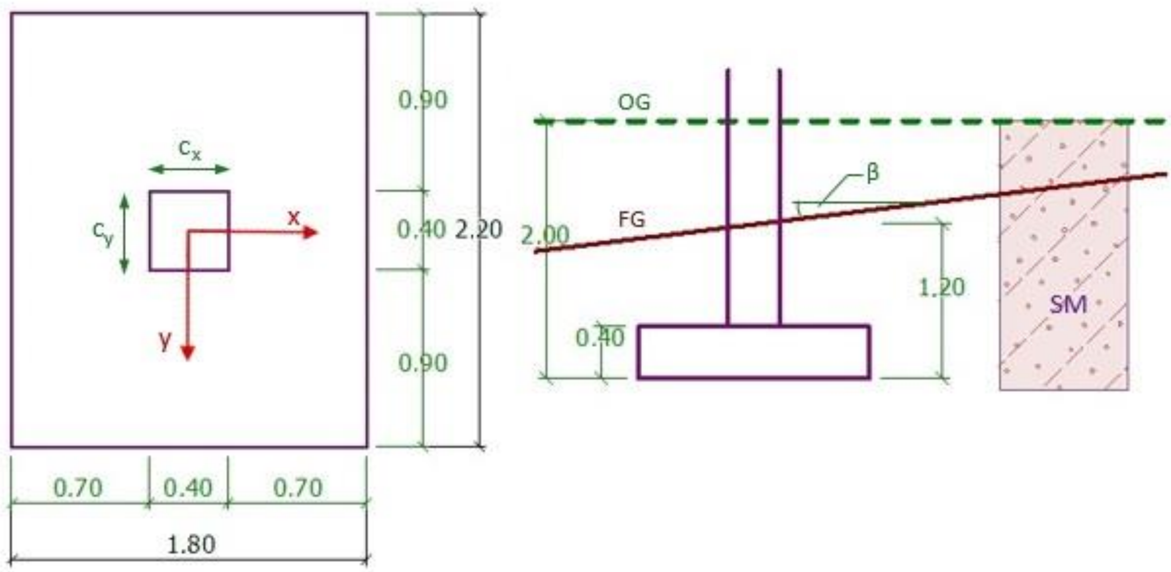

Figure 1 Top view and cross-section in x of spread footing

Soil	Unit weight γ [kN/m^3]	Saturated unit weight γ_{sat} [kN/m^3]	Angle of internal friction φ_{ef} [°]	Cohesion of soil c_{ef} [kPa]	Poisson's ration ν [-]
SM	17.50	17.50	31.50	0.00	0.35

Table 1 Soil properties – characteristic effective values

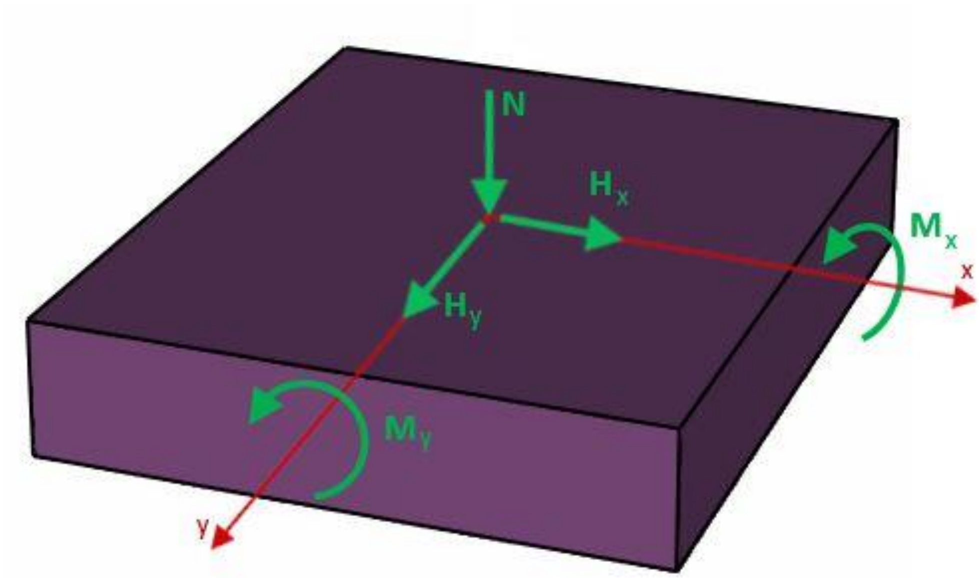

Figure 2 Load on the spread footing

Design value	$N\,[kN]$	$H_x\,[kN]$	$H_y\,[kN]$	$M_x\,[kNm]$	$M_y\,[kNm]$
Yes	910.00	0.00	120.00	200.00	0.00

Table 2 Values of the load

1. The Calculation with Self-Weight acting Favourably

Calculation of the effective area of the spread footing. An eccentric load acts on the spread footing. Therefore, the effective area of the spread footing has to be calculated.

- Characteristic weight of the spread footing:
  $G = b \cdot h \cdot t \cdot \gamma_c = 1.80 \cdot 2.20 \cdot 0.40 \cdot 23.00 = 36.432\ kN$

- Characteristic weight of the overburden:

$$Z = \left[(d-t)\cdot b\cdot h-(d-t)\cdot c_x\cdot c_y\right]\cdot\gamma_z = \left[(1.20-0.40)\cdot 1.80\cdot 2.20-(1.20-0.40)\cdot 0.40\cdot 0.40\right]\cdot 20.00$$

$$Z = 60.80\ kN$$

- Calculation of the load eccentricity at the footing bottom $(\gamma_G = 1.00)$:

$$e_y = \frac{M_x + H_y\cdot t}{N + Z\cdot\gamma_G + G\cdot\gamma_G} = \frac{200.00 + 120.00\cdot 0.40}{910.00 + 60.80\cdot 1.00 + 36.432\cdot 1.00} = 0.246\ m$$

In the program, eccentricity is calculated as a ratio.

$$e_{y,ratio} = \frac{e_y}{l} = \frac{0.246}{2.200} = 0.112$$

- Verification of the load eccentricity:

$e_{x,ratio} = 0.000 \quad\le\quad 0.333 = e_{alw}$ - max. eccentricity in direction of the base length

$e_{y,ratio} = 0.112 \quad\le\quad 0.333 = e_{alw}$ - max. eccentricity in direction of the base width

$e_{t,ratio} = \sqrt{e^2_{x.ratio} + e^2_{y,ratio}}$ - max. overall eccentricity

$e_{t,ratio} = 0.112 \quad\le\quad 0.333 = e_{alw}$

The eccentricity of the load is **SATISFACTORY.**

Result from the GEO5 – Spread Footing program:

$e_{x,ratio} = 0.000 \quad\le\quad 0.333 = e_{alw}$

$e_{y,ratio} = 0.112 \quad\le\quad 0.333 = e_{alw}$

$e_{t,ratio} = 0.112 \quad\le\quad 0.333 = e_{alw}$, **SATISFACTORY**

- Calculation of the effective area of the spread footing (a rectangular shape of the effective area is assumed):

$$b_{eff} = b = 1.80\ m$$

$$l_{eff} = l - 2\cdot e = 2.20 - 2\cdot 0.246 = 1.708\ m$$

$$A_{eff} = b_{eff}\cdot l_{eff} = 1.80\cdot 1.708 = 3.074\ m^2$$

Calculation of the vertical bearing capacity.

A standard analysis proposed by J. Brinch Hansen is used.

- Coefficients of bearing capacity:

$$N_q = tg^2\left(45+\frac{\varphi}{2}\right)\cdot e^{\pi\cdot tg(\varphi)} = tg^2\left(45+\frac{31.5}{2}\right)\cdot e^{\pi\cdot tg(31.5)} = 21.861$$

$$N_c = (N_d - 1)\cdot \cot g(\varphi) = (21.861-1)\cdot \cot g(31.5) = 34.042$$

$$N_\gamma = 1.5\cdot (N_d - 1)\cdot tg(\varphi) = 1{,}5\cdot(21.861-1)\cdot tg(31.5) = 19.175$$

- Coefficients of the shape of the foundation:

Due to the eccentricity, the following situation occurred: $l_{eff} < b_{eff}$. Therefore, $b = l_{eff}$ and $l = b_{eff}$ were used.

$$s_q = 1+\frac{b}{l}\cdot\sin(\varphi) = 1+\frac{1.708}{1.800}\cdot\sin(31.5) = 1.496$$

$$s_c = 1+0.2\cdot\frac{b}{l} = 1+0.2\cdot\frac{1.708}{1.800} = 1.190$$

$$s_\gamma = 1-0.3\cdot\frac{b}{l} = 1-0.3\cdot\frac{1.708}{1.800} = 0.715$$

- Coefficients of influence of the foundation depth:

$$d_q = 1+0.1\cdot\sqrt{\frac{d}{b}\cdot\sin(2\cdot\varphi)} = 1+0.1\cdot\sqrt{\frac{1.200}{1.708}\cdot\sin(2\cdot 31.5)} = 1.079$$

$$d_c = 1+0.1\cdot\sqrt{\frac{d}{b}} = 1+0.1\cdot\sqrt{\frac{1.200}{1.708}} = 1.084$$

$$d_\gamma = 1.000$$

- Coefficients of influence of the slope load:

δ - angle of deviation of the resultant force from the vertical direction

$$\delta = arctg\left(\frac{H_y}{N+Z\cdot\gamma_G+G\cdot\gamma_G}\right) = arctg\left(\frac{120}{910+60.80\cdot 1.00+36.432\cdot 1.00}\right) = 6.794°$$

$$i_q = i_c = i_\gamma = (1-tg(\delta))^2 = (1-tg(6.794))^2 = 0.776$$

- Coefficients of slope of the footing bottom:

α - slope of the footing bottom

$\alpha = 0.000°$

$b_q = (1-\alpha\cdot tg(\varphi))^2 = (1-0\cdot tg(31.5))^2 = 1.000$

$$b_c = b_q - \frac{(1-b_q)}{N_c} \cdot tg(\varphi) = 1 - \frac{(1-1)}{34.042} \cdot tg(31.5) = 1.000$$

$$b_\gamma = b_q = 1.000$$

- Coefficients of influence of the slope of the terrain:
  β - slope of the terrain
  $\beta = 7.000°$

$$g_q = g_\gamma = (1 - 0.5 \cdot tg(\beta))^5 = (1 - 0.5 \cdot tg(7.0))^5 = 0.728$$

$$g_c = 1 - \frac{2 \cdot \beta}{\pi + 2} = 1 - \frac{2 \cdot \left(7.000 \cdot \frac{\pi}{180}\right)}{\pi + 2} = 0.952$$

- Equivalent uniform load accounting for the influence of the foundation depth:
  $q_0 = \gamma_1 \cdot d = 17.5 \cdot 1.20 = 21.00\ kPa$

- Calculation of the vertical bearing capacity and its reduction by coefficient $\gamma_{RV} = 1.40$:

$$R_d = c \cdot N_c \cdot s_c \cdot d_c \cdot i_c \cdot b_c \cdot g_c + q_0 \cdot N_q \cdot s_q \cdot d_q \cdot i_q \cdot b_q \cdot g_q + \gamma \cdot \frac{b}{2} \cdot N_\gamma \cdot s_\gamma \cdot d_\gamma \cdot i_\gamma \cdot c_\gamma \cdot b_\gamma \cdot g_\gamma$$

$$R_d = 0.0 + 21.0 \cdot 21.861 \cdot 1.496 \cdot 1.079 \cdot 0.776 \cdot 1.0 \cdot 0.728 + 17.50 \cdot \frac{1.708}{2} \cdot 19.175 \cdot 0.715 \cdot 1.0 \cdot 0.776 \cdot 1.0 \cdot 0.728$$

$$R_d = 0.0 + 418.635 + 115.753$$

$$R_d = 534.388\ kPa$$

$$\frac{R_d}{\gamma_{RV}} = \frac{534.388}{1.40} = 381.706\ kPa$$

Result from the GEO5 – Spread Footing program: $R_d = 381.92\ kPa$

- Extreme contact stress at the footing bottom:

$$\sigma = \frac{N + Z \cdot \gamma_G + G \cdot \gamma_G}{A_{eff}} = \frac{910.00 + 60.80 \cdot 1.00 + 36.432 \cdot 1.00}{3.074} = 327.662\ kPa$$

Result from the GEO5 – Spread Footing program: $\sigma = 327.70\ kPa$

- Usage:

$$V_u = \frac{\sigma}{R_d} \cdot 100 = \frac{327.662}{381.706} \cdot 100 = 85.8\,\%$$, SATISFACTORY

Result from the GEO5 – Spread Footing program: $V_u = 85.8\,\%$ **, SATISFACTORY**

Calculation of the horizontal bearing capacity.

The earth resistance is taken as pressure at rest.

- Coefficient of earth pressure at rest in a drained soil:
  $K_0 = 1 - \sin(\varphi_d) = 1 - \sin(31.5) = 0.4775$

- Pressure at rest in the axis on the upper and lower edge of the spread footing:
  $\sigma_{01} = \gamma \cdot (d - t) \cdot K_0 = 17.50 \cdot (1.20 - 0.40) \cdot 0.4775 = 6.6850\ kPa$

  $\sigma_{z2} = \gamma \cdot d \cdot K_0 = 17.50 \cdot 1.20 \cdot 0.4775 = 10.0275\ kPa$

- Value of earth resistance:
  $$S_{pd} = \left[\frac{1}{2} \cdot (\sigma_{02} - \sigma_{01}) \cdot t + \sigma_{01} \cdot t\right] \cdot b = \left[\frac{1}{2} \cdot (10.0275 - 6.6850) \cdot 0.40 + 6.6850 \cdot 0.40\right] \cdot 1.80 = 6.017\ kN$$

- Resultant vertical force on the footing bottom:
  $Q = V + Z + G = 910.00 + 60.80 + 36.432 = 1007.232\ kN$

- Horizontal bearing capacity and its reduction by coefficient $\gamma_{RH} = 1.10$:
  $a_d \cdot A_{eff}$ - is excluded for drained conditions (according to EC 1997)

$$R_{dh} = \frac{Q \cdot tg(\varphi_d) + a_d \cdot A_{eff} + S_{pd}}{\gamma_{RH}}$$

$$R_{dh} = \frac{Q \cdot tg(\varphi_d) + S_{pd}}{\gamma_{RH}} = \frac{1007.232 \cdot tg(31.5) + 6.017}{1.10} = 566.591\ kN$$

Result from the GEO5 – Spread Footing program: $R_{dh} = 566.59\ kPa$

- Usage:
  $$V_u = \frac{H}{R_{dh}} \cdot 100 = \frac{120.000}{566.591} \cdot 100 = 21.2\,\%$$, SATISFACTORY

Result from the GEO5 – Spread Footing program: $V_u = 21.2\,\%$ **, SATISFACTORY**

2. The Calculation with Self-Weight Acting Unfavourably

Calculation of the effective area. The characteristic weights of the spread footing and the overburden are the same as in the calculation with self-weight in favor.

- Characteristic weight of the spread footing:
  $G = 36.432\ kN$

- Characteristic weight of the overburden:
  $Z = 60.80\ kN$

- Calculation of the load eccentricity at the footing bottom $(\gamma_G = 1.35)$:
  $$e_y = \frac{M_x + H_y \cdot t}{N + Z \cdot \gamma_G + G \cdot \gamma_G} = \frac{200.00 + 120.00 \cdot 0.40}{910.00 + 60.80 \cdot 1.35 + 36.432 \cdot 1.35} = 0.238\ m$$

  In the program, eccentricity is calculated as a ratio.
  $$e_{y,pom} = \frac{e_y}{l} = \frac{0.238}{2.200} = 0.108$$

- Verification of the load eccentricity:
  $e_{x,pom} = 0.000 \quad \leq \quad 0.333 = e_{alw}$ - max. eccentricity in direction of the base length

  $e_{y,pom} = 0.108 \quad \leq \quad 0.333 = e_{alw}$ - max. eccentricity in direction of the base width

  $e_{t,pom} = \sqrt{e_{x.pom}^2 + e_{y,pom}^2}$ - max. overall eccentricity

  $e_{t,pom} = 0.108 \quad \leq \quad 0.333 = e_{alw}$

  The eccentricity of the load is **SATISFACTORY.**

  Result from the GEO5 – Spread Footing program:
  The GEO5 – Spread Footing program shows the verification of a load that causes greater eccentricity. In this case a load with self-weight in favour is verified.

- Calculation of the effective area of the spread footing (a rectangular shape of the effective area is assumed):
  $b_{eff} = b = 1.80\ m$

  $l_{eff} = l - 2 \cdot e = 2.20 - 2 \cdot 0.238 = 1.724\ m$

  $A_{eff} = b_{eff} \cdot l_{eff} = 1.80 \cdot 1.724 = 3.103\ m^2$

Calculation of the vertical bearing capacity.

- Coefficients of bearing capacity:

$$N_q = tg^2\left(45+\frac{\varphi}{2}\right)\cdot e^{\pi\cdot tg(\varphi)} = tg^2\left(45+\frac{31.5}{2}\right)\cdot e^{\pi\cdot tg(31.5)} = 21.861$$

$$N_c = (N_d - 1)\cdot \cot g(\varphi) = (21.861-1)\cdot \cot g(31.5) = 34.042$$

$$N_\gamma = 1.5\cdot(N_d - 1)\cdot tg(\varphi) = 1.5\cdot(21.861-1)\cdot tg(31.5) = 19.175$$

- Coefficients of the shape of the foundation:

Due to the eccentricity, the following situation occurred: $l_{eff} < b_{eff}$. Therefore, $b = l_{eff}$ and $l = b_{eff}$ were used.

$$s_q = 1+\frac{b}{l}\cdot\sin(\varphi) = 1+\frac{1.724}{1.800}\cdot\sin(31.5) = 1.500$$

$$s_c = 1+0.2\cdot\frac{b}{l} = 1+0.2\cdot\frac{1.724}{1.800} = 1.192$$

$$s_\gamma = 1-0.3\cdot\frac{b}{l} = 1-0.3\cdot\frac{1.724}{1.800} = 0.713$$

- Coefficients of influence of the foundation depth:

$$d_q = 1+0.1\cdot\sqrt{\frac{d}{b}\cdot\sin(2\cdot\varphi)} = 1+0.1\cdot\sqrt{\frac{1.200}{1.724}\cdot\sin(2\cdot 31.5)} = 1.079$$

$$d_c = 1+0.1\cdot\sqrt{\frac{d}{b}} = 1+0.1\cdot\sqrt{\frac{1.200}{1.724}} = 1.083$$

$$d_\gamma = 1.000$$

- Coefficients of influence of the slope of the load:

δ - angle of deviation of the resultant force from the vertical direction

$$\delta = arctg\left(\frac{H_y}{N+Z\cdot\gamma_G+G\cdot\gamma_G}\right) = arctg\left(\frac{120}{910+60.80\cdot 1.35+36.432\cdot 1.35}\right) = 6.574^\circ$$

$$i_q = i_c = i_\gamma = (1-tg(\delta))^2 = (1-tg(6.574))^2 = 0.783$$

- Coefficients of the slope of the footing bottom:

α -slope of the footing bottom

$\alpha = 0.000^\circ$

$$b_q = (1 - \alpha \cdot tg(\varphi))^2 = (1 - 0 \cdot tg(31.5))^2 = 1.000$$

$$b_c = b_q - \frac{(1 - b_q)}{N_c} \cdot tg(\varphi) = 1 - \frac{(1-1)}{34.042} \cdot tg(31.5) = 1.000$$

$$b_\gamma = b_q = 1.000$$

- Coefficients of influence of the slope of the terrain:
  β - slope of the terrain
  $\beta = 7.000°$

$$g_q = g_\gamma = (1 - 0.5 \cdot tg(\beta))^5 = (1 - 0.5 \cdot tg(7.0))^5 = 0.728$$

$$g_c = 1 - \frac{2 \cdot \beta}{\pi + 2} = 1 - \frac{2 \cdot \left(7.000 \cdot \frac{\pi}{180}\right)}{\pi + 2} = 0.952$$

- Equivalent uniform load accounting for the influence of the foundation depth:
  $q_0 = \gamma_1 \cdot d = 17.50 \cdot 1.20 = 21.00\ kPa$

- Calculation of the vertical bearing capacity and its reduction by coefficient $\gamma_{RV} = 1.40$:

$$R_d = c \cdot N_c \cdot s_c \cdot d_c \cdot i_c \cdot b_c \cdot g_c + q_0 \cdot N_q \cdot s_q \cdot d_q \cdot i_q \cdot b_q \cdot g_q + \gamma \cdot \frac{b}{2} \cdot N_\gamma \cdot s_\gamma \cdot d_\gamma \cdot i_\gamma \cdot c_\gamma \cdot b_\gamma \cdot g_\gamma$$

$$R_d = 0.0 + 21.0 \cdot 21.861 \cdot 1.500 \cdot 1.079 \cdot 0.783 \cdot 1.0 \cdot 0.728 + 17.50 \cdot \frac{1.724}{2} \cdot 19.175 \cdot 0.713 \cdot 1.0 \cdot 0.783 \cdot 1.0 \cdot 0.728$$

$$R_d = 0.0 + 423.541 + 117.561$$

$$R_d = 541.102\ kPa$$

$$\frac{R_d}{\gamma_{RV}} = \frac{541.102}{1.40} = 386.501\ kPa$$

Result from the GEO5 – Spread Footing program: $R_d = 386.61\ kPa$

- Extreme contact stress at the footing bottom:

$$\sigma = \frac{N + Z \cdot \gamma_G + G \cdot \gamma_G}{A_{eff}} = \frac{910.00 + 60.80 \cdot 1.35 + 36.432 \cdot 1.35}{3.103} = 335.567\ kPa$$

Result from the GEO5 – Spread Footing program: $\sigma = 335.61\ kPa$

- Usage:

$$V_u = \frac{\sigma}{R_d} \cdot 100 = \frac{335.567}{386.501} \cdot 100 = 86.8\ \%$$, SATISFACTORY

Calculation of the parameters of a slip surface below the foundation. The parameters of a slip surface are calculated according to Prandtl's theory.

- Depth of the slip surface:

$$z_{sp} = \frac{b}{2} \cdot \frac{\cos(\varphi)}{\cos\left(45 + \frac{\varphi}{2}\right)} \cdot e^{\left(\frac{\pi}{4} + \frac{arc(\varphi)}{2}\right) \cdot tg(\varphi)} = \frac{1.80}{2} \cdot \frac{\cos(31.5)}{\cos\left(45 + \frac{31.5}{2}\right)} \cdot e^{\left(\frac{\pi}{4} + \frac{arc(31.5)}{2}\right) \cdot tg(31.5)} = 3.008\ m$$

Result from the GEO5 – Spread Footing program: $z_{sp} = 3.01\ m$

- Length of the slip surface:

$$l_{sp} = \frac{b}{2} \cdot \left[1 + 2 \cdot tg\left(45 + \frac{\varphi}{2}\right) \cdot e^{\frac{\pi}{2} \cdot tg(\varphi)}\right] = \frac{1.80}{2} \cdot \left[1 + 2 \cdot tg\left(45 + \frac{31.5}{2}\right) \cdot e^{\frac{\pi}{2} \cdot tg(31.5)}\right] = 9.316\ m$$

Result from the GEO5 – Spread Footing program: $z_{sp} = 9.31\ m$