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Md. Mahmud Sazzad
Rajshahi University of Engineering & Technology

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MD. MONIRUZZAMAN MONI, MD. MAHMUD SAZZAD
Department of Civil Engineering, Rajshahi University of Engineering & Technology, Bangladesh
Email: mmmruet@gmail.com, mmsruet@gmail.com

Abstract: This article demonstrates a finite element approach to analyze the response of slopes of homogeneous and layered soil with shear strength reduction (SSR) technique. Slope failure may occur owing to a reduction of shear strength with increasing plastic strains induced by loading. The present approach allows this failure process and analysis is carried out using Mohr-Coulomb and Drucker-Prager yield criteria in which the shear strength parameters are reduced. The effect of mesh on the factor of safety of slope is investigated. To assess the reliability of the numerical result for slope stability analysis by the finite element SSR technique, its comparison with the well known conventional methods namely Bishop Method (1955), Fellenius Method (1936) and Spencer Method (1967) is shown for homogeneous and layered soil. Good agreement is found between the conventional limit equilibrium methods (LEM) and finite element method (FEM). It is noted that FEM gives a bit higher factor of safety than LEM. Besides, Drucker-Prager model gives higher factor of safety than that of Mohr-Coulomb model regardless of the position of surcharge and slope angles for both the slopes of homogeneous and layered soil. The factor of safety increases when the distance of surcharge increases from the crest of the slope up to a certain level and beyond that, the effect of surcharge remains constant for slopes of homogeneous and layered soil. The failure surfaces for both LEM and FEM have also been assessed.

Keywords: *Slope stability, Surcharge, Finite element method, Shear strength reduction technique, Conventional method, Factor of safety*

1. Introduction:

Slope stability analysis is one of the most important areas of interest in geotechnical engineering. There are a lot of engineering structures which require foundation systems to be placed near an existing slope such as bridge abutment, tower footings, basement construction of high rise building, etc. In construction areas, slope may fail due to heavy rainfall, increase in ground water table and change in stress condition. Similarly, natural slopes that have been stable for many years may suddenly fail due to change in topography, external forces, loss of shear strength, and weathering (Abramson et al. 2002). Therefore, it is a common challenge to both researchers and professionals to analyse the stability of slopes and evaluate the certainty of the factor of safety. Lin and Cao (2012) conducted the effect of shear strength parameters, cohesion and internal friction angle, on the stability of slope through theoretical derivation and limit equilibrium method. In their study, changes in the factor of safety of slope and slip surface were investigated. Namdar (2010) presented the three-wedge method for stability analysis of slope. The influence of root trees on slope stability was studied and different factors like geometry and gradient, geologic materials, stratigraphy, hydrology and the local effects on the shore process were analyzed as well. Cala and Flisiak (2003) performed many simulations for isotropic and homogeneous slope using shear strength reduction (SSR) technique and limit equilibrium methods (LEM). The influence of elastic properties (Young's modulus $= E$, Poison's ratio $= \nu$) on slope stability analysis were investigated and it was noted that elastic properties negligibly influenced the factor of safety of slope. In their study, the effect of slope angle and slope height was carried out as well and the results obtained by SSR technique were compared with that of LEM.

Duncan (1996) proposed that the stability and deformation of slope can be analyzed by finite element method (FEM). Griffiths and Lane (1999) discussed several examples of FEM based slope stability analysis by comparing with other solution methods. Zhang et al. (2010) evaluated the channel slope stability of the East Route of the South-North Water Diversion Project, China. Typical channel cross section in Sanding Province was evaluated using SSR-FEM. To describe the stress-strain relationship of the soils, Duncan-Chang nonlinear constitutive model was employed. The factor of safety calculated by strength reduction method was compared with LEM. He and Zhang (2012) described the stability analysis of a homogeneous slope and showed that the equivalent area circle Drucker-Prager yield criterion was suitable for the stability analysis of slope.

For the stability analysis of slope, factor of safety can be calculated by different methods. Over the past four decades, numerical analyses have been conducted mainly through conventional LEM. These methods are statically indeterminate and require pre-assumptions to determine the factor of safety. The application of LEM is limited to the simple shape of slope and not available for complex geometries. By contrast, the numerical methods such as FEM have been widely used over the last two decades. In FEM, any assumption in advance of the failure shape and location of the failure surface are not necessary (Griffiths and Lane 1999).

Even though many researches have been carried out for the stability analysis of slope by LEM and FEM for homogeneous soil but a few studies for layered soil with surcharge on the stability analysis of slope have been reported in the literature. Consequently, the objectives of the present study are: (i) to investigate the effect of mesh on the factor of safety of slope, (ii) to compare the FEM based analysis result with that of LEM, (iii) to evaluate the effect of soil layer on the stability of slope, (iv) to assess the effect of the position of surcharge and (v) to examine the mode of slope failure obtained from FEM analysis and compare the same with LEM.

2. Methods of Evaluating the Factor of Safety of Slope:

2.1 Limit Equilibrium Methods:

Several limit equilibrium methods were available in the literature to determine the factor of safety of slope. Some of the well-known and widely used LEM methods are Bishop method (1955), Fellenius method (1936) and Spencer method (1967). The main disadvantage of conventional LEM is that it requires pre-assumptions to complete the solution. The solution in LEM is simple; however, it can be inadequate in case the slope fails by complex mechanism such as internal deformation, brittle failure, etc. A summary of several limit equilibrium methods and their assumptions are presented in Table 1.

Table 1*: Summery of limit equilibrium methods (SLOPE/W 2004; Abramson et al. 2002)*

Methods	Moment Equilibrium	Force Equilibrium	Shape of Slip surface	Interslice Normal (E)	Interslice Shear (T)	Assumptions for T and E
Ordinary or Fellenius	Yes	No	Circular	No	No	No interslice forces
Bishop's Simplified	Yes	No	Circular	Yes	No	The side forces are Horizontal
Janbu's Simplified	No	Yes	Any shape	Yes	No	The side forces are Horizontal
Janbu's Generalised	Yes (by slice)	Yes	Any shape	Yes	Yes	Applied line of thrust and moment equilibrium of slice
Lowe-Karafiath	No	Yes	Any shape	Yes	Yes	Average of ground surface and slice base inclination
Corps of Engineers	No	Yes	Any shape	Yes	Yes	Inclination of ground surface at top of slice
Sarma	Yes	Yes	Any shape	Yes	Yes	Interslice shear $T = ch + E\ tan\phi$
Spencer	Yes	Yes	Any shape	Yes	Yes	Constant inclination $T = tan\theta\ E$
Morgenstern-Price	Yes	Yes	Any shape	Yes	Yes	Defined by $f(x), T = f(x).\lambda.E$

2.2 Finite Element Method:

FEM is a powerful numerical tool for solving many engineering problems and mathematical physics. Due to rapid development of computer technology, FEM has gained increasing popularity over the traditional methods in geotechnical engineering. Generally, there are two approaches to analyze the stability of slope using FEM. One approach is to increase the gravity load of soil element and the second approach is to reduce the strength characteristics of the soil mass usually called Shear Strength Reduction (SSR) technique. The SSR technique is adopted in the present study by using a powerful FEM based software GEO5 (2014). In SSR technique, it is assumed that slope materials have elasto-plastic behavior. The SSR is based on the progressive reduction of soil strength parameters, ϕ and c until the failure of slope occurs. The factored shear-strength parameters c_f and θ_f are given as follows:

$$c_f = \frac{c}{F_s}, \qquad (1)$$

$$\phi_f = \arctan\left(\frac{\tan\phi}{F_s}\right), \qquad (2)$$

where F_s is a strength reduction factor. For details of SSR technique, readers are referred to Griffiths and Lane (1999).

2.3 Geometric Model of Slope:

A number of problems for different slope angles are solved in the present paper. Fig. 1 shows the geometric model of a slope of homogeneous soil while Fig. 2 presents the geometric model of a slope of layered soil. In both the models, β represents the slope angle, x is distance of surcharge from the crest of the slope and W is the width of surcharge. Here, β and x are variables and W is constant (2 meter). In Fig. 2, h and t represents the distance of a thin soil layer from the top of the slope and thickness of thin layer, respectively. Here, h is variable and t is constant. The finite element models of slopes of homogeneous and layered soil are shown in Figs. 3 and 4, respectively.

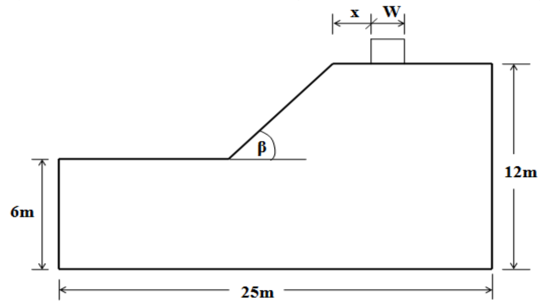


***Figure 1**: Geometric model of a slope of homogeneous soil*

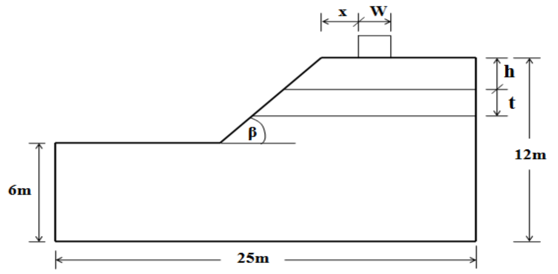


Figure 2: Geometric model of a slope of layered soil

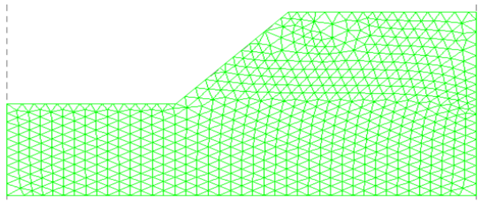


Figure 3: Geometry and mesh for a slope of homogeneous soil

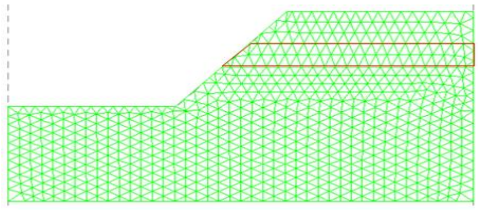


Figure 4: Geometry and mesh for a slope of layered soil

2.4 Material Properties:

The properties of soil used in the present study are presented in Table 2. Two types of soils are considered. Soil-1 is used in the analysis of a slope of homogeneous soil whereas both Soil-1 and Soi-2 are used in the analysis of a slope of layered soil. Soil-2 is used for thin layer of slope of layered soil.

***Table 2**: Properties of soil considered in the present study*

Material	Unit weight (kN/m^3)	Friction angle (degree)	Cohesion (kN/m^2)	Modulus of elasticity (MN/m^2)	Poisson's ratio	Dilation angle (degree)
Soil-1	20	18	10	8	0.3	0
Soil-2	20	10	6	8	0.3	0

2.5 Loading and Boundary Conditions:

In all cases, it is assumed that there is no external load other than the gravitational force (i.e. body force). Two different geometric models are used in this study. In both the models (Figs. 1 and 2), the geometric boundaries are horizontally constrained on the left and right sides and completely fixed at the bottom of the geometry.

3. Numerical Modeling of Slopes:

The effect of mesh is studied in this section. Two types of meshes: (i) 6-node triangular elements and (ii) mixed mesh consisting of triangular and quadrilateral elements are used in this study. The mesh is determined by the selection of approximate global size. A slope stability benchmark example has been considered in this study. The benchmark problem considers a slope of homogeneous soil. Fig. 5 shows the geometry of slope used for the benchmark problem. The slope of the benchmark problem is inclined at an angle of 29.74 degree to the horizontal.

In this example, ten different mesh configurations are used to study the effects of mesh on the factor of safety of slope. In this analysis, approximate global size is varied from 0.5 to 1.5 for generating the meshes. The value of 0.5 produces finer mesh than that of 1.5. Ten analysis cases for the benchmark model (Fig. 5) using FEM have been considered and they are summarized in Table 3. The benchmark models for the analysis cases 1 and 2 are depicted in Figs. 6 and 7, respectively as examples. In Table 3, 'TE6' indicates 6-node triangular element and 'Mixed' indicates mixing of triangular and quadrilateral elements.

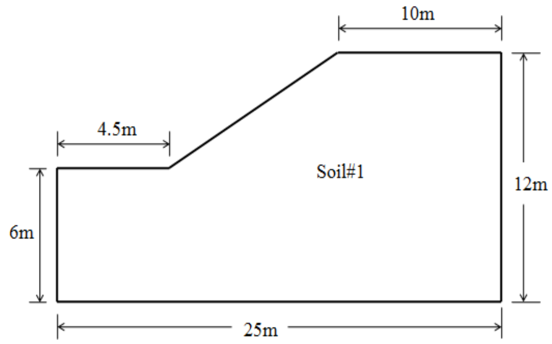


***Figure 5**: Geometry of the benchmark model*

***Table 3**: Different analysis cases for the benchmark model by FEM*

Analysis case	Mesh type	Element number	Node number
Analysis 1	TE6	411	694
Analysis 2	Mixed	295	572
Analysis 3	TE6	794	1391
Analysis 4	Mixed	541	1138
Analysis 5	TE6	1156	2061
Analysis 6	Mixed	776	1705
Analysis 7	TE6	1941	3544
Analysis 8	Mixed	1202	2791
Analysis 9	TE6	2655	4906
Analysis 10	Mixed	1610	3847

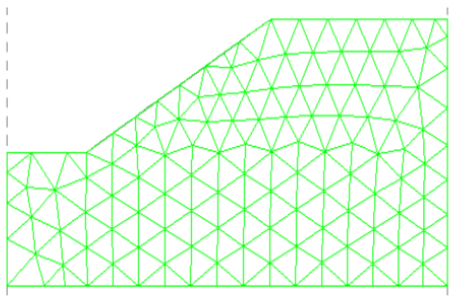


***Figure 6**: 6-node triangular elements with approximate global size of 1.5*

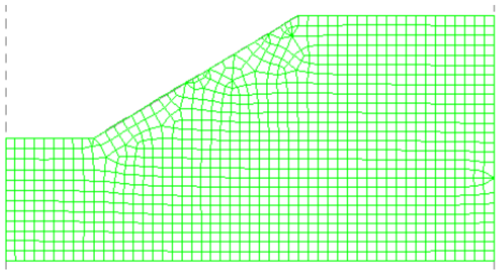


***Figure 7:** Mixed mesh with approximate global size of 0.5*

4. Numerical Analysis by LEM and FEM:

In the present study, a number of numerical analyses have been performed by the software GEO5 (2014). Bishop method (1955), Fellenius method (1936) and Spencer method (1967) are used for limit equilibrium analysis. Mohr-Coulomb failure criterion and Drucker-Prager yield criterion are used in finite element analysis. For LEM, the geometric model is incorporated in the GEO5 (2014) software and the properties of soil are assigned for the specified interface. In the analysis stage, a slip surface is added. The slip surface may be circular or polygonal. In this paper, circular slip surface is used. After assigning all properties and slip circle, optimization method is selected as analysis type. Finally, a surcharge is added on the terrain of slope and analysis is carried out. For the stability analysis of slopes using FEM, the first step is to set the project parameters. Plane strain project type is selected. Later, analysis type is set. The geometric model is incorporated in the GEO5 (2014) same as LEM. After incorporating the model, the properties of soil are assigned for the specified interface. For FEM analysis, meshes are generated and a strip surcharge is added on the terrain of slope. Finally, analysis is performed using the SSR technique (Griffiths and Lane 1999).

5. Results and Discussions:

5.1 Effect of Mesh:

The effects of mesh on the factor of safety for ten different mesh configurations are shown in Table 4. It is noted that the computed factor of safety ranges from 1.37 to 1.54. The finer mesh gives more conservative results than the coarser mesh. The factor of safety using 6-node triangular elements is very close to that of mixed mesh.

From the analysis of different cases, it can be concluded that the factor of safety varies up to an approximate active element number of 1200 for both TE6 and mixed mesh and it remains constant for active element number larger than 1200. Note that this result is valid only for the geometry used in this study and it may vary depending on the size of the geometry, selection of global size and user's experience.

***Table 4**: Factor of safety for different analysis cases*

Analysis case	Method	Mesh type	Factor of safety
Analysis 1	FEM	TE6	1.50
Analysis 2	FEM	Mixed	1.54
Analysis 3	FEM	TE6	1.46
Analysis 4	FEM	Mixed	1.50
Analysis 5	FEM	TE6	1.44
Analysis 6	FEM	Mixed	1.46
Analysis 7	FEM	TE6	1.44
Analysis 8	FEM	Mixed	1.44
Analysis 9	FEM	TE6	1.44
Analysis 10	FEM	Mixed	1.44
Bishop	LEM	-	1.45
Fellenius	LEM	-	1.37
Spencer	LEM	-	1.44

5.2 Slope of Homogeneous Soil:

Table 5 shows the comparison of factor of safety between FEM and LEM with surcharge (x/W =1.5 and β =45°.) Note that, factor of safety by Fellenius method (1936) is lower than that by Bishop (1955) and Spencer (1967). Note also that Drucker-Prager model shows greater factor of safety than that of Mohr-Coulomb model. LEM results are very close to that of FEM for a slope of homogeneous soil. This indicates the effectiveness of FEM in analyzing the stability of slopes. Figs. 8 to 10 depict the effect of the variation of x/W on the factor of safety of a slope of homogeneous soil with surcharge using both FEM and LEM for slope angles of 30°, 45°and 60°, respectively. Several interesting findings are noticed. Factor of safety gradually increases as x/W increases regardless of material models, slope angle or LEM used. Interesting point is that the gradual increase of x/W becomes constant at a certain value of x/W. For β =30°, the factor of safety reaches a constant value at x/W =3 for Mohr-Coulomb and Drucker-Prager model and x/W =2 for Bishop (1955) and Fellenius (1936) methods and x/W =2.5 for Spencer (1967). For β =45°, factor of safety reaches a constant value at x/W =2 for Bishop (1955) method and x/W =2.5 for Mohr-Coulomb, Drucker-Prager, Fellenius (1936) and Spencer (1967) method. For β =60°, factor of safety reaches a constant value at x/W =2 for Mohr-Coulomb and Drucker-Prager model and x/W =2.5 for Bishop (1955) Fellenius (1936) and Spencer (1967) method. The above data depict that the range where the factor of safety becomes constant lies between 2 to 3, regardless of material models, slope angles or methods of analysis. Fig. 11 shows the effect of slope angle on the factor of safety of slope with LEM and FEM when x/W =0. It is depicted that factor of safety decreases as the slope angle increases regardless of material model, LEM or FEM.

***Table 5**: Comparison of factor of safety between FEM and LEM (x/W =1.5, β =45°)*

Material models and methods	Factor of safety
Drucker-Prager	1.05
Mohr-Coulomb	1.00
Bishop	1.02
Fellenius	0.98
Spencer	1.02

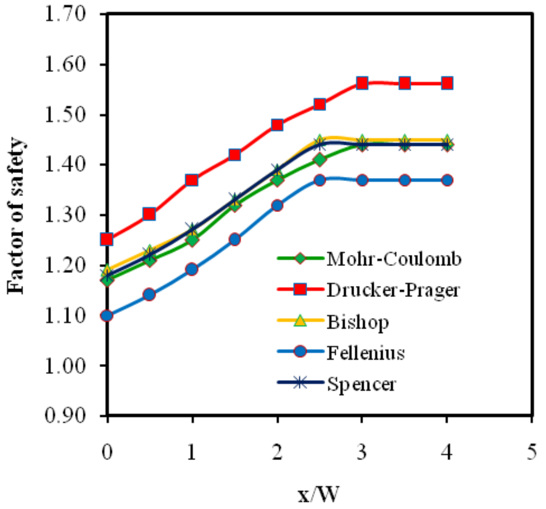


***Figure 8**: Effect of the variation of x/W on the factor of safety of slope considering both LEM and FEM (β = 30°)*

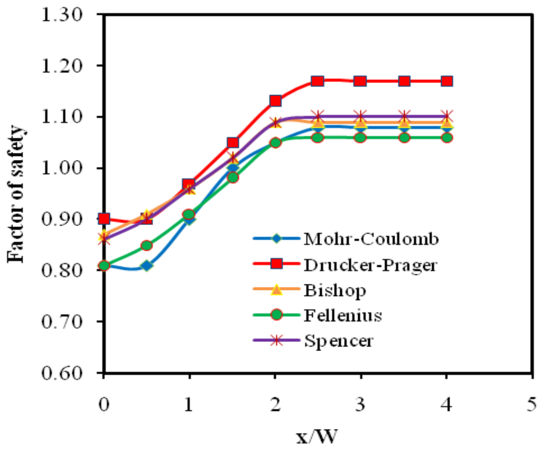


***Figure 9:** Effect of the variation of x/W on the factor of safety of slope considering both LEM and FEM (β = 45°)*

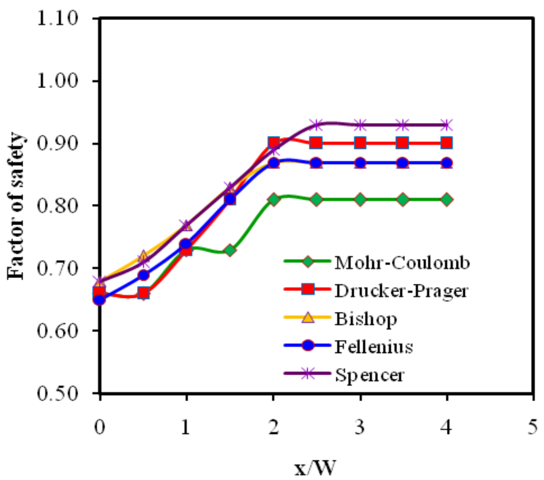


***Figure 10**: Effect of the variation of x/W on the factor of safety of slope considering both LEM and FEM (β = 60°)*

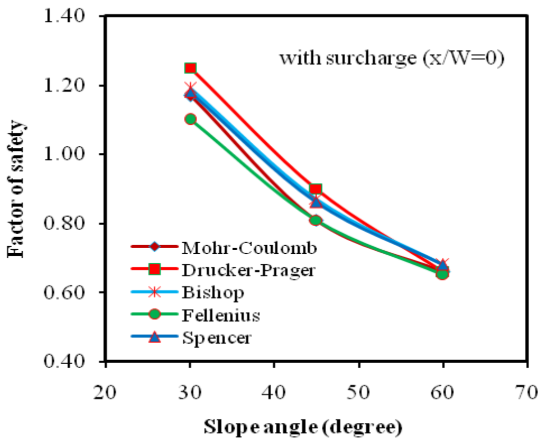


***Figure 11**: Effect of slope angle on the factor of safety of slope for LEM and FEM*

Fig. 12 shows the contours of the equivalent plastic strain (a measure of the amount of permanent strain in an engineering body) for a slope of homogeneous soil (β =30°) by FEM while Fig. 13 presents the failure of a slope of homogeneous soil (β =30°) with surcharge using LEM. Similarly, Fig. 14 shows the contours of the equivalent plastic strain for a slope of homogeneous soil (β =45°) by FEM while Fig. 15 presents the failure of a slope of homogeneous soil (β =45°) with surcharge using LEM. It is obvious from Figs. 12 to 15 that slip surfaces obtained from FEM are localized deeper than LEM irrespective of slope angles when homogeneous soil is considered.

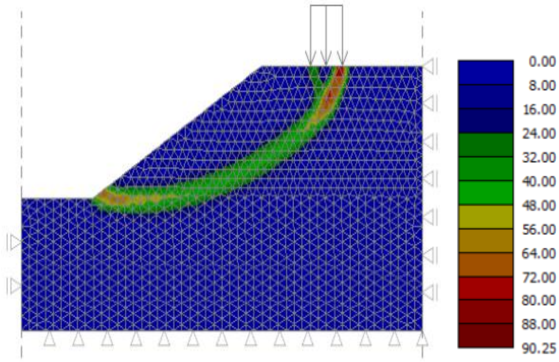


***Figure 12:** Contours of the equivalent plastic strain for a slope of homogeneous soil (β=30°) with surcharge by FEM*

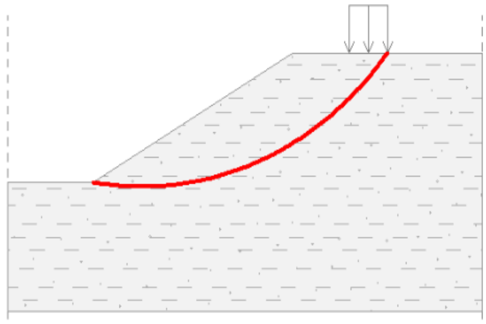


***Figure 13**: Failure of a slope of homogeneous soil (β =30°) with surcharge using LEM*

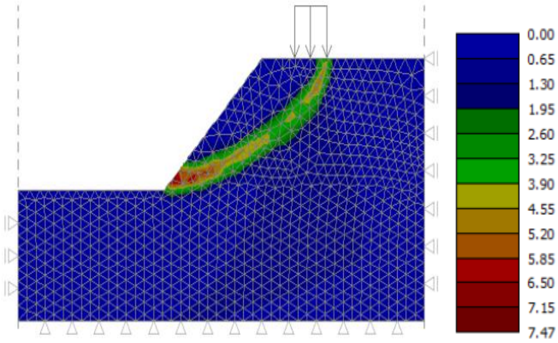


***Figure 14**: Contours of the equivalent plastic strain for a slope of homogeneous soil (β =45°) with surcharge by FEM*

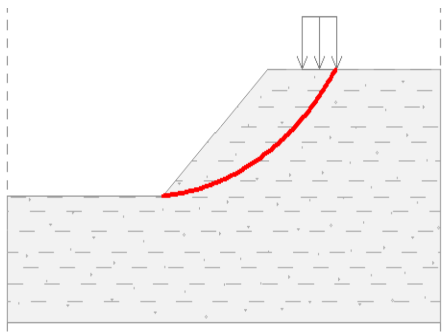


***Figure 15:** Failure of a slope of homogeneous soil (β = 45°) with surcharge using LEM*

5.3 Slope of Layered Soil:

The effect of the variation of the position of a thin weak layer (thickness= t) from top of the slope on the factor of safety is discussed in this section. The thickness of the thin weak layer is set to 2 meter and the position of this layer is varied. First, the weak layer is considered at the top of the slope (i.e. $h/t=0$). Fig. 16 shows the effect of the position of surcharge on the factor of safety of slope of layered soil by LEM and FEM for $h/t=0$ and β =30°. Similar results are depicted in Figs. 17, 18 and 19 for $h/t=1$, 2 and 3, respectively and β =30°. Drucker-Prager model depicts the highest factor of safety compared to Mohr-Coulomb model regardless of the position of the thin weak soil layer. When $h/t=0,1$, FEM results considering Mohr-Coulomb model and LEM give almost same results. However, difference is apparent as h/t keeps increasing (Figs. 18 and 19). Similar results are noticed for slope angles of 45° and 60°.

Fig. 20 shows the contours of the equivalent plastic strain for a slope of layered soil by FEM (β = 45° and $h/t=0$) while Fig. 21 depicts the failure of a slope of layered soil by LEM (β = 45° and $h/t=0$). Note that when weak soil layer is located at the top of the slope, the failure of slope occurs only in the weak portion of the soil. As the position of the weak soil layer changes from the top of the slope (i.e. h increases), the failure line (i.e. slip surfaces) extends to the end of the weak layer (Figs. 22-27). When weak soil layer is located at the foundation layer of the slope (Figs. 26-27), slip surfaces also extend to the foundation layer.

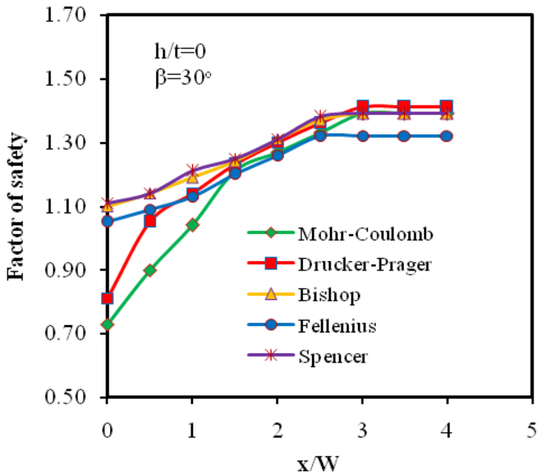


Figure 16: *Effect of the variation of x/W on the factor of safety of slope ($h/t=0$ and β = 30°)*

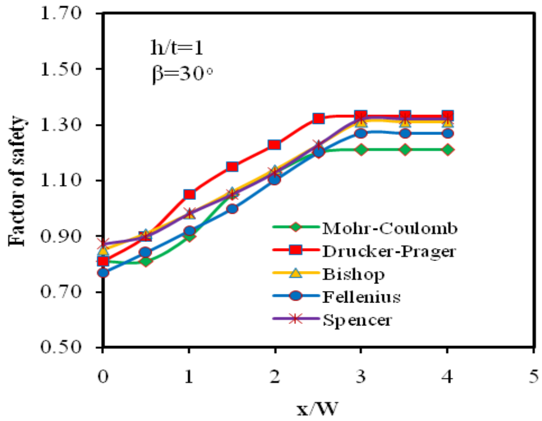


Figure 17*: Effect of the variation of x/W on the factor of safety of slope ($h/t=1$, β =30°)*

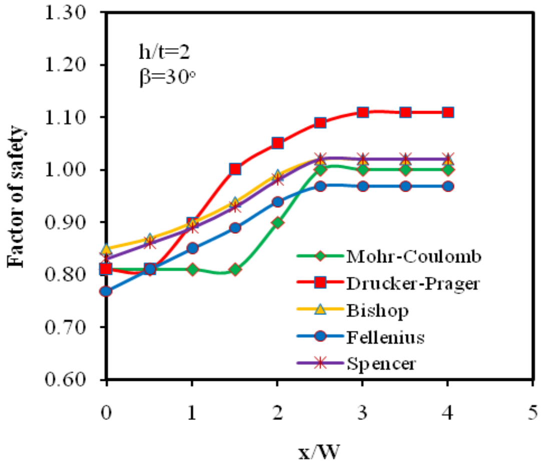


Figure 18*: Effect of the variation of x/W on the factor of safety of slope ($h/t=2$ and β =30°)*

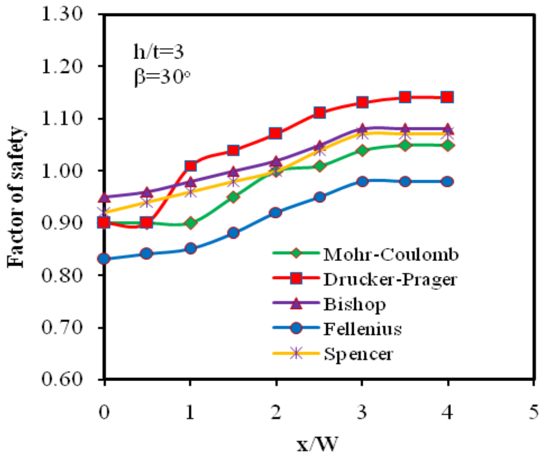


Figure 19*: Effect of the variation of x/W on the factor of safety of slope ($h/t=3$ and β =30°)*

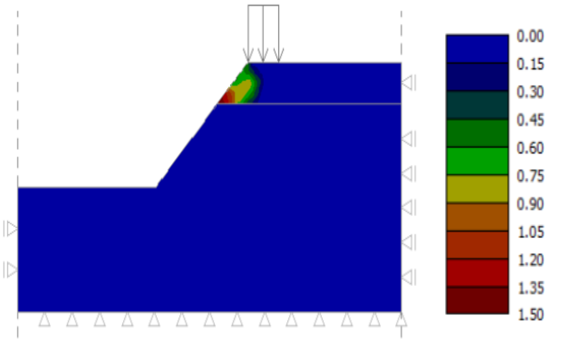


Figure 20: *Contours of the equivalent plastic strain for slope of layered soil with surcharge by FEM ($\beta =$ 45° and $h/t = 0$)*

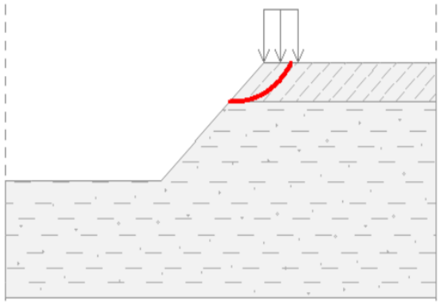


Figure 21: *Failure of a slope of layered soil with surcharge by LEM ($\beta = 45°$ and $h/t = 0$)*

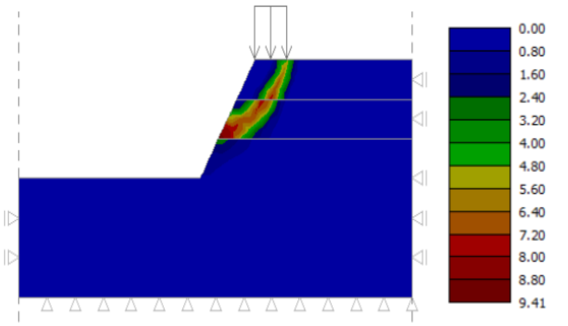


Figure 22: *Contours of the equivalent plastic strain for slope of layered soil with surcharge by FEM ($\beta =$ 60° and $h/t = 1$)*

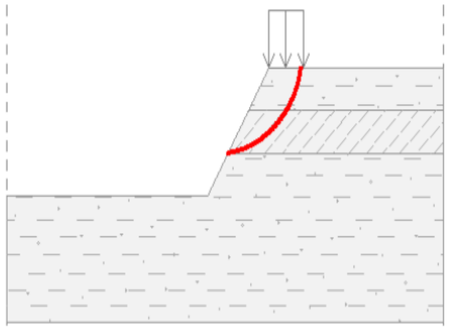


Figure 23: *Failure of a slope of layered soil with surcharge by LEM ($\beta = 60°$ and $h/t = 1$)*

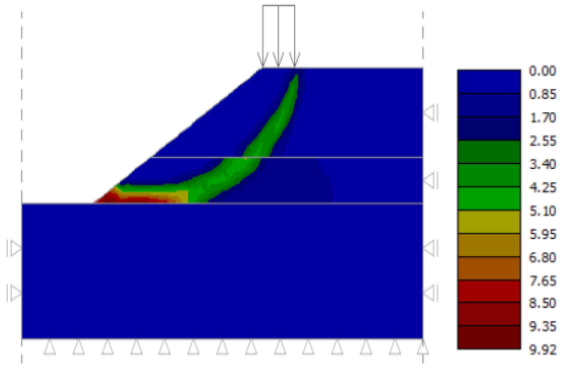


Figure 24: *Contours of the equivalent plastic strain for slope of layered soil with surcharge by FEM ($\beta =$ 30° and $h/t = 2$)*

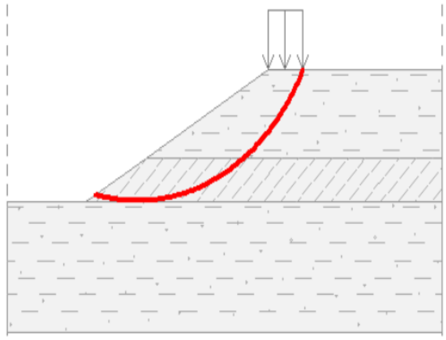


Figure 25: *Failure of a slope of layered soil with surcharge by LEM ($\beta = 30°$ and $h/t = 2$)*

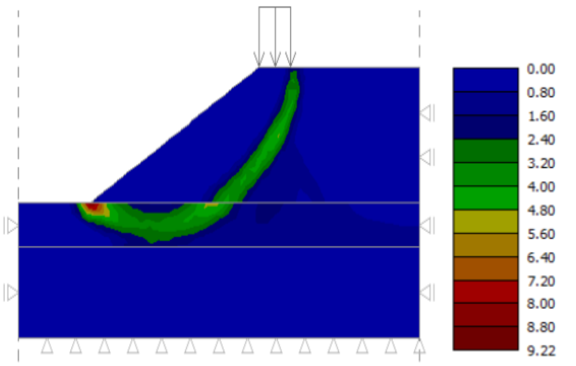


Figure 26: *Contours of the equivalent plastic strain for slope of layered soil with surcharge by FEM ($\beta =$ 30° and $h/t = 3$)*

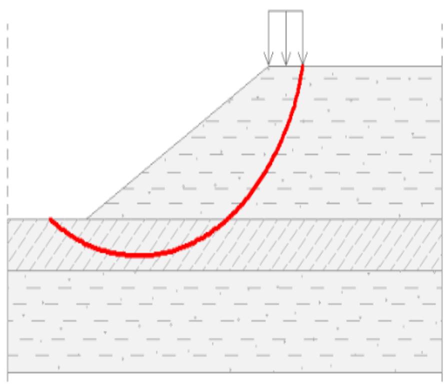
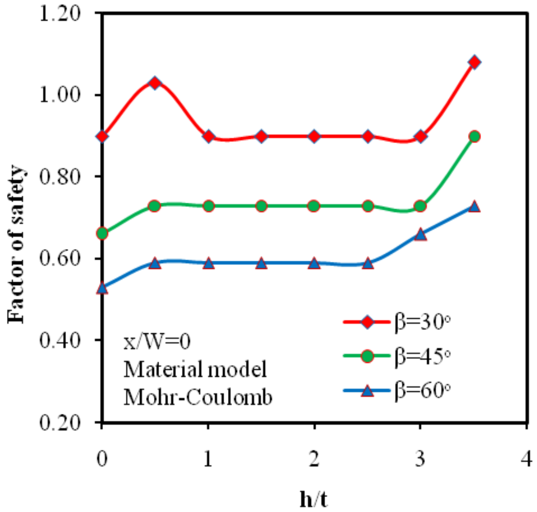
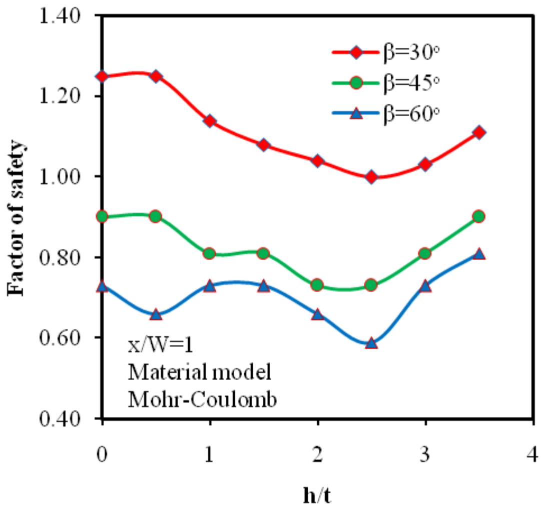


Figure 27: *Failure of a slope of layered soil with surcharge by LEM ($\beta = 30°$ and $h/t = 3$)*

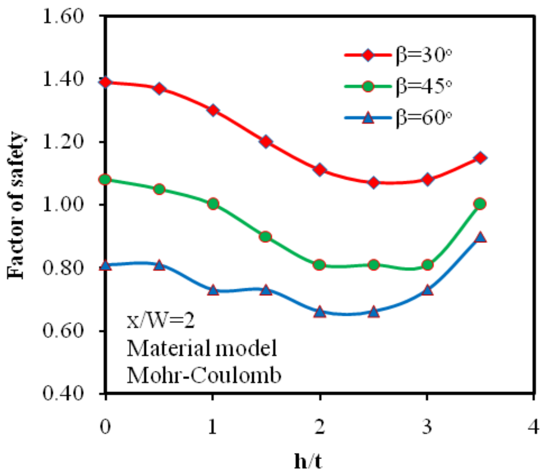
Fig. 28 depicts the effect of soil layering on the factor of safety of slope for various slope angles. Factor of safety decreases with the increase of h/t ratio up to a certain level and beyond that level, it starts increasing again.



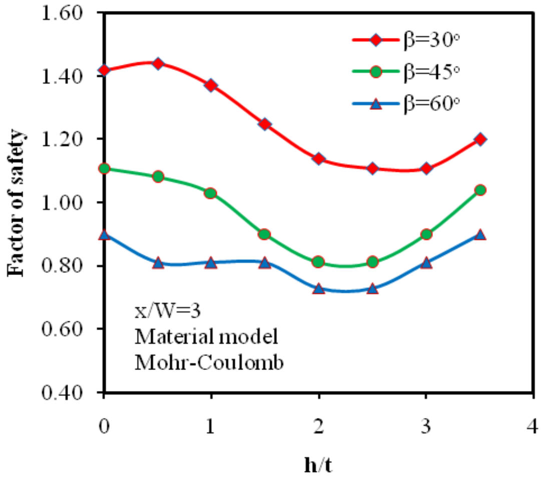
(a)



(b)



(c)



(d)

***Figure 28**: Effect of soil layer on the factor of safety of slope for various slope angles*

6. Conclusions:

A detailed numerical investigation is carried out to study the responses of slopes of homogeneous and layered soil with surcharge by shear strength reduction (SSR) technique. The effect is mesh is studied and the factor of safety obtained by FEM is compared to that by LEM. The effect of layered soil on the stability of slope by LEM and FEM is evaluated for the variation of the position of surcharge, slope angles and material models. The mode of failure has also been studied and discussed. The major findings of the study are summarized as follows:

i. The number of finite element mesh has effect on the factor of safety of slope. Factor of safety varies up to an approximate active element number of 1200 for the geometry considered in the present study for both TE6 and mixed mesh and beyond that, factor of safety remains constant.

ii. The factor of safety computed by Fellenius method (1936) gives a bit lower value than that of Bishop (1955) and Spencer methods (1967) irrespective of the position of surcharge and slope angles considering both the slopes of homogeneous and layered soil.

iii. The factor of safety considering the Mohr-Coulomb model depicts lower value than that of Drucker-Prager model irrespective of the position of surcharge and slope angles for both the slopes of homogeneous and layered soil.

iv. Factor of safety is a function of the position of surcharge from the crest of slope for a certain level only. Beyond that level, it has no influence of the factor of safety of slopes of homogeneous and layered soil.

v. The factor of safety of slope of layered soil decreases with the increase of h/t up to a certain value and beyond that, the factor of safety increases again.

vi. FEM depicts deeper localization of slip surface than LEM for both the slope of homogeneous and layered soil.

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