Verification Analysis of the Gravity Wall

Program: Gravity Wall
File: Demo_vm_en_01.gtz

In this verification manual you will find hand-made verification analysis calculations of gravity wall in permanent and seismic design situations. The results of the hand-made calculations are compared with the results from the GEO5 – Gravity Wall program.

Terms of Reference:

In Figure 1, an example of a gravity wall with inclined footing bottom in 1:10 inclination is shown. The earth body is comprised of two soil layers and the terrain is adjusted in 1:10 inclination. The top layer of the earth body (depth 1.5 m) is formed of sandy silt (MS). The lower layer of the earth body is formed of clayey sand (SC), which is at the front face of the wall too. The groundwater table is in the depth of 1.5 m behind the wall and 3.7 m in front of the wall. The properties of soils (effective values) are in Table 1. The gravity wall is made from plain concrete C20/25 with unit weight $\gamma = 23$ kN/m$^3$. A verification analysis of the wall is performed with the help of the theory of limit states.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image.png}
\caption{Construction of gravity wall – dimensions}
\end{figure}
Soil Unit weight $\gamma$ [kN/m$^3$] Saturated unit weight $\gamma_{sat}$ [kN/m$^3$] Angle of internal friction $\phi_{ef}$ [°] Cohesion of soil $c_{ef}$ [kPa] Angle of friction struc.-soil $\delta$ [°] Poisson’s ratio $\nu$ [-]

<table>
<thead>
<tr>
<th>Soil</th>
<th>$\gamma$</th>
<th>$\gamma_{sat}$</th>
<th>$\phi_{ef}$</th>
<th>$c_{ef}$</th>
<th>$\delta$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS</td>
<td>18.00</td>
<td>20.00</td>
<td>26.50</td>
<td>12.00</td>
<td>15.00</td>
<td>0.35</td>
</tr>
<tr>
<td>SC</td>
<td>18.50</td>
<td>20.50</td>
<td>27.00</td>
<td>8.00</td>
<td>15.00</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Table 1 Soil properties – characteristic effective values

The angle of friction and cohesion enter the first phase of the calculation as design values and that’s why the soil parameters from Table 1 are reduced with coefficients $\gamma_{m\phi} = 1.1$ and $\gamma_{mc} = 1.4$. The design values used in calculation are in Table 2.

<table>
<thead>
<tr>
<th>Soil</th>
<th>Angle of internal friction $\phi_{ef, d}$ [°]</th>
<th>Cohesion of soil $c_{ef, d}$ [kPa]</th>
<th>Angle of friction struc.-soil $\delta_{d}$ [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS</td>
<td>24.091</td>
<td>8.571</td>
<td>13.636</td>
</tr>
<tr>
<td>SC</td>
<td>24.545</td>
<td>5.714</td>
<td>13.636</td>
</tr>
</tbody>
</table>

Table 2 Soil properties – design values

1. The First Stage- Permanent Design Situation

Verification of the Whole Wall

Calculation of the weight force and the centroid of the wall. The wall is divided into 5 parts, which are shown in Figure 1. Parts 4 and 5 are under the groundwater table, therefore the unit weight of concrete is reduced by the unit weight of water $\gamma_w = 10$ kN/m$^3$. Table 3 shows the dimensions of the parts of the wall, their weight forces and centroids.

<table>
<thead>
<tr>
<th>Part</th>
<th>height $h_i$ [m]</th>
<th>width $b_i$ [m]</th>
<th>Area $A_i$ [m$^2$]</th>
<th>Part weight $\gamma_i$ [kN/m$^3$]</th>
<th>Weight force $W_i$ [kN/m]</th>
<th>Point of action $x_i$ [m]</th>
<th>$z_i$ [m]</th>
<th>$G_i \cdot x_i$</th>
<th>$G_i \cdot z_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.500</td>
<td>0.700</td>
<td>2.450</td>
<td>23</td>
<td>56.350</td>
<td>1.950</td>
<td>-2.550</td>
<td>109.883</td>
<td>-143.693</td>
</tr>
<tr>
<td>2</td>
<td>3.500</td>
<td>0.700</td>
<td>1.225</td>
<td>23</td>
<td>28.175</td>
<td>1.367</td>
<td>-1.967</td>
<td>38.506</td>
<td>-55.411</td>
</tr>
<tr>
<td>3</td>
<td>0.200</td>
<td>2.300</td>
<td>0.460</td>
<td>23</td>
<td>10.580</td>
<td>1.150</td>
<td>-0.700</td>
<td>12.167</td>
<td>-7.406</td>
</tr>
<tr>
<td>4</td>
<td>0.600</td>
<td>2.300</td>
<td>1.380</td>
<td>13</td>
<td>17.940</td>
<td>1.150</td>
<td>-0.300</td>
<td>20.631</td>
<td>-5.382</td>
</tr>
<tr>
<td>5</td>
<td>0.230</td>
<td>2.300</td>
<td>0.265</td>
<td>13</td>
<td>3.439</td>
<td>1.533</td>
<td>0.077</td>
<td>5.273</td>
<td>0.264</td>
</tr>
<tr>
<td>Total</td>
<td>116.484</td>
<td>-</td>
<td>-</td>
<td>186.460</td>
<td>-211.628</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3 Dimensions, weight force and centroids of the individual blocks
Centroid of the construction:

\[ x_i = \frac{\sum_{i=1}^{5} W_i \cdot x_i}{\sum_{i=1}^{5} W_i} = \frac{186.460}{116.484} = 1.601 \, m \]

\[ z_i = \frac{\sum_{i=1}^{5} W_i \cdot z_i}{\sum_{i=1}^{5} W_i} = -211.628 \div 116.484 = -1.817 \, m \]

**Calculation of the front face resistance.** The depth of soil in front of the wall is 0.6 m. Pressure at rest is considered.

- **Hydraulic gradient:**
  \[ i = \frac{h_w}{d_d + d_u} = \frac{3.7 - 1.5}{3.03 + 0.6} = 0.606 \]

- **Coefficient of earth pressure at rest:**
  (For cohesive soils the Terzaghi formula for computing of the coefficient of earth pressure at rest \( K_r \) is used)
  \[ K_r = \frac{v}{1 - v} = \frac{0.35}{1 - 0.35} = 0.538 \]

- **Unit weight of soil in the area of ascending flow:**
  \[ \gamma = \gamma_{sat} - \gamma_w - \gamma_w \cdot i = 20.5 - 10.0 - 10 \cdot 0.606 = 4.439 \, kN/ m^3 \]

- **Vertical normal effective stress \( \sigma_z \) in the footing bottom:**
  \[ \sigma_z = \gamma \cdot h = 4.439 \cdot 0.6 = 2.663 \, kPa \]

- **Pressure at rest in the footing bottom:**
  \[ \sigma_r = \sigma_z \cdot K_r = 2.663 \cdot 0.538 = 1.433 \, kPa \]

- **Resultant force of stress at rest \( S_r \):**
  (Resultant force \( S_r \) acts only in horizontal direction, therefore \( S_r = S_{rx} \) and \( S_{rz} = 0 \))
  \[ S_r = \frac{1}{2} \sigma_r \cdot h = \frac{1}{2} \cdot 1.433 \cdot 0.6 = 0.430 \, kN/ m \]
- Point of action of the resultant force $S_r$:
  $$x = 0.000 \, m$$
  $$z = -\frac{1}{3} \cdot h = -\frac{1}{3} \cdot 0.6 = -0.200 \, m$$

**Calculation of the active pressure.** There are two layers of soils in the area, where we solve the active pressure. The structure is therefore divided into two sections, in both of which the geostatic pressure $\sigma_z$, the active earth pressure $\sigma_a$ and the resultant forces $S_{a1}$ and $S_{a2}$ are calculated. The active earth pressure is calculated using Coulomb’s theory.

![Figure 2 Geostatic pressure $\sigma_z$ and active pressure $\sigma_a$](image)

- Coefficients of active earth pressure in both sections:
  $(\alpha = 0^\circ$ - back face inclination of the structure, $\beta \neq 0^\circ$-inclination of the terrain; design values of soils from Table 2 are used in the calculation)

  $$K_a = \frac{\cos^2(\varphi - \alpha)}{\cos^2(\alpha) \cdot \cos(\alpha + \delta) \left( 1 + \frac{\sin(\varphi + \delta) \cdot \sin(\varphi - \beta)}{\cos(\alpha + \delta) \cdot \cos(\alpha - \beta)} \right)^2}$$
$K_{ac}$ - coefficient of active earth pressure due to cohesion

$$K_{ac} = \frac{\cos(\varphi) \cdot \cos(\beta) \cdot \cos(\delta - \alpha) \cdot \left[ 1 + \tan(-\alpha) \cdot \tan(\beta) \right]}{1 + \sin(\varphi + \delta - \alpha - \beta)} \cdot \frac{1}{\cos(\delta + \alpha)}$$

**Calculation for the first section:**

$$\beta_1 = \beta = \arctan\left(\frac{1}{10}\right) = 5.711^\circ$$

$$K_{a1} = \frac{\cos^2(24.091 - 0)}{\cos^2(0) \cdot \cos(0 + 13.636) \cdot \left[ 1 + \frac{\sin(24.091 + 13.636) \cdot \sin(24.091 - 5.711)}{\cos(0 + 13.636) \cdot \cos(0 - 5.711)} \right]^2} = 0.4097$$

$$K_{ac1} = \frac{\cos(24.091) \cdot \cos(5.711) \cdot \cos(13.636 - 0) \cdot \left[ 1 + \tan(-0) \cdot \tan(5.711) \right]}{1 + \sin(24.091 + 13.636 - 0 - 5.711)} \cdot \frac{1}{\cos(13.636 + 0)} = 0.5936$$

**Calculation for the second section:**

$$\beta_2 = \arctan\left(\frac{\gamma \cdot \tan(\beta)}{\gamma_i}\right) = \arctan\left(\frac{18.0 \cdot \tan(5.711)}{18.5}\right) = 5.557^\circ$$

$$K_{a2} = \frac{\cos^2(24.545 - 0)}{\cos^2(0) \cdot \cos(0 + 13.636) \cdot \left[ 1 + \frac{\sin(24.545 + 13.636) \cdot \sin(24.545 - 5.557)}{\cos(0 + 13.636) \cdot \cos(0 - 5.557)} \right]^2} = 0.4016$$

$$K_{ac2} = \frac{\cos(24.545) \cdot \cos(5.557) \cdot \cos(13.636 - 0) \cdot \left[ 1 + \tan(-0) \cdot \tan(5.557) \right]}{1 + \sin(24.545 + 13.636 - 0 - 5.557)} \cdot \frac{1}{\cos(13.636 + 0)} = 0.5882$$

- Unit weight of soil SC in the area of descending flow:

  $$\gamma_2 = \gamma_{sat} - \gamma_w + \gamma \cdot i = 20.5 - 10.0 + 10 \cdot 0.606 = 16.561 \text{ kN/m}^3$$

- Vertical geostatic pressure $\sigma_g$ in two sections:

  $$\sigma_{g1} = \gamma_1 \cdot h_1 = 18.0 \cdot 1.5 = 27.000 \text{ kPa}$$

  $$\sigma_{g2} = \gamma_1 \cdot h_1 + \gamma_2 \cdot h_2 = 18.0 \cdot 1.5 + 16.561 \cdot 3.03 = 77.180 \text{ kPa}$$

- Calculation of high in first layer of soil MS, where the active earth pressure is neutral:

  $$h_0 = \frac{2 \cdot c_{sf,d1} \cdot K_{ac1}}{\gamma_1 \cdot K_{a1}} = \frac{2 \cdot 8.571 \cdot 0.5936}{18.0 \cdot 0.4097} = 1.380 \text{ m}$$

- Active earth pressure $\sigma_a$ in two sections:

  $$\sigma_{a1} = \sigma_{c1} \cdot K_{a1} - 2 \cdot c_{sf,d1} \cdot K_{ac1} = 27.00 \cdot 0.4097 - 2 \cdot 8.571 \cdot 0.5936 = 0.886 \text{ kPa}$$

5
\[ \sigma_{a2a} = \sigma_{z1} \cdot K_{a2} - 2 \cdot c_{ef,a2} \cdot K_{ac2} = 27.00 \cdot 0.4016 - 2 \cdot 5.714 \cdot 0.5882 = 4.121 \text{ kPa} \]

\[ \sigma_{a2b} = \sigma_{z2} \cdot K_{a2} - 2 \cdot c_{ef,a2} \cdot K_{ac2} = 77.18 \cdot 0.4016 - 2 \cdot 5.714 \cdot 0.5882 = 24.274 \text{ kPa} \]

- **Resultant forces of active earth pressure** \( S_{a1}, S_{a2} \) and vertical and horizontal components:

  \[ S_{a1} = \frac{1}{2} \cdot \sigma_{a1} \cdot (h_1 - h_0) = \frac{1}{2} \cdot 0.886 \cdot (1.500 - 1.380) = 0.053 \text{ kN/m} \]

  \[ S_{a1,x} = S_{a1} \cdot \cos(\delta) = 0.053 \cdot \cos(13.636) = 0.052 \text{ kN/m} \]

  \[ S_{a1,z} = S_{a1} \cdot \sin(\delta) = 0.053 \cdot \sin(13.636) = 0.013 \text{ kN/m} \]

  \[ S_{a2} = \frac{1}{2} \cdot (\sigma_{a2b} - \sigma_{a2a}) \cdot h_2 + \sigma_{a2a} \cdot h_2 = \frac{1}{2} \cdot (24.274 - 4.121) \cdot 3.03 + 4.121 \cdot 3.03 = 43.018 \text{ kN/m} \]

  \[ S_{a2,x} = S_{a2} \cdot \cos(\delta) = 43.018 \cdot \cos(13.636) = 41.806 \text{ kN/m} \]

  \[ S_{a2,z} = S_{a2} \cdot \sin(\delta) = 43.018 \cdot \sin(13.636) = 10.142 \text{ kN/m} \]

- **Points of action of resultant forces** \( S_{a1} \) and \( S_{a2} \):

  \[ x_1 = 2.300 \text{ m} \]

  \[ z_1 = -0.8 - 2.0 - \frac{1.5 - 1.38}{3} = -2.840 \text{ m} \]

  \[ x_2 = 2.300 \text{ m} \]

  \[ z_2 = \frac{4.121 \cdot 3.03 \cdot \left(-\frac{3.03}{2} + 0.23\right) + (24.274 - 4.121) \cdot \frac{3.03}{2} \cdot \left(-\frac{3.03}{3} + 0.23\right)}{4.121 \cdot 3.03 + (24.274 - 4.121) \cdot \frac{3.03}{2}} = -0.927 \text{ m} \]

- **Total resultant force of active earth pressure** \( S_a \):

  \[ S_{ax} = S_{a1,x} + S_{a2,x} = 0.052 + 41.806 = 41.858 \text{ kN/m} \]

  \[ S_{az} = S_{a1,z} + S_{a2,z} = 0.013 + 10.142 = 10.155 \text{ kN/m} \]

  \[ S_a = \sqrt{S_{ax}^2 + S_{az}^2} = \sqrt{41.858^2 + 10.155^2} = 43.072 \text{ kN/m} \]

- **Point of action of total resultant force:**
Calculation of the water pressure. The heel of the structure is sunken in a permeable subsoil, which allows free water flow below the structure. Therefore, the hydrodynamic pressure must be considered and its resultant force is calculated as shown in Figure 3. The area of the hydrodynamic pressure is divided into two sections.

- Horizontal water pressure \( \sigma_w \) at interface of section 1 and section 2 (depth 3.7 m):
  \[
  \sigma_w = \gamma_w \cdot (3.7 - 1.5) = 10 \cdot 2.2 = 22.000 \text{ kPa}
  \]

- Resultant force of water pressure \( S_w \) in two sections:
  \[
  S_{w1} = \frac{1}{2} \cdot \sigma_w \cdot h_{w1} = \frac{1}{2} \cdot 22.000 \cdot 2.2 = 24.200 \text{ kN/m}
  \]
\[ S_{w2} = \frac{1}{2} \cdot \sigma_w \cdot h_{w2} = \frac{1}{2} \cdot 22.000 \cdot 0.83 = 9.130 \text{ kN/m} \]

- Points of action of resultant forces:
  \[ x_1 = 2.300 \text{ m} \]
  \[ z_1 = -0.6 - \frac{2.2}{3} = -1.333 \text{ m} \]
  \[ x_2 = 2.300 \text{ m} \]
  \[ z_2 = -0.6 + \left( \frac{0.6 + 0.23}{3} \right) = -0.323 \text{ m} \]

- Total resultant force of water pressure \( S_w \):
  \[ S_w = \sum_{i=1}^{2} S_{w,i} = 24.200 + 9.130 = 33.330 \text{ kN/m} \]

- Point of action of resultant force \( S_w \):
  \[ x_w = 2.300 \text{ m} \]
  \[ z_w = \frac{\sum_{i=1}^{2} S_{wi} \cdot z_i}{\sum_{i=1}^{2} S_{wi}} = \frac{24.2 \cdot (-1.333) + 9.13 \cdot (-0.323)}{33.33} = -1.056 \text{ m} \]

**Checking for overturning stability.** The moments calculated in the analysis rotate about the origin of the coordinate system (left bottom corner of the structure). Resisting moment \( M_{res} \) and overturning moment \( M_{ovr} \) are calculated for verification.

- Calculation of resisting moment \( M_{res} \) and its reduction by coefficient \( \gamma_S = 1.1 \):
  \[ M_{res} = W \cdot r_1 + S_{az} \cdot r_2 = 116.484 \cdot 1.601 + 10.155 \cdot 2.3 = 209.847 \text{ kNm/m} \]
  \[ \frac{M_{res}}{\gamma_S} = \frac{209.847}{1.1} = 190.770 \text{ kNm/m} \]

  **Result from the GEO5 – Gravity Wall program:** \( M_{res} = 190.74 \text{ kNm/m} \)

- Calculation of overturning moment \( M_{ovr} \):
  \[ M_{ovr} = -0.430 \cdot 0.2 + 41.858 \cdot 0.929 + 33.33 \cdot 1.056 = 73.997 \text{ kNm/m} \]

  **Result from the GEO5 – Gravity Wall program:** \( M_{ovr} = 74.02 \text{ kNm/m} \)
• **Usage:**

\[
V_u = \frac{M_{\text{act}}}{M_{\text{res}}} \cdot 100 = \frac{73.997}{190.770} \cdot 100 = 38.8\% \text{, SATISFACTORY}
\]

**Result from the GEO5 – Gravity Wall program:** \( V_u = 38.8\% \text{, SATISFACTORY} \)

**Checking for slip.** Slip in the inclined footing bottom (Figure 4).

![Figure 4 Forces acting in the footing bottom](image)

- **Total vertical and horizontal forces** \( F_{\text{ver}} \) and \( F_{\text{hor}} \):

\[
F_{\text{ver}} = 116.484 + 10.155 = 126.639 \text{ kN/m}
\]

\[
F_{\text{hor}} = -0.43 + 41.858 + 33.33 = 74.758 \text{ kN/m}
\]

- **Normal force** \( N \):

\[
\alpha_b = 5.711^\circ
\]

\[
N = F_{\text{ver}} \cdot \cos(\alpha_b) + F_{\text{hor}} \cdot \sin(\alpha_b) = 126.639 \cdot \cos(5.711) + 74.758 \cdot \sin(5.711) = 133.450 \text{ kN/m}
\]

- **Shear force** \( T \):

\[
T = -F_{\text{ver}} \cdot \sin(\alpha_b) + F_{\text{hor}} \cdot \cos(\alpha_b) = -126.639 \cdot \sin(5.711) + 74.758 \cdot \cos(5.711) = 61.785 \text{ kN/m}
\]

- **Eccentricity of normal force:**

\( d \) - inclined width of footing bottom
In the program, eccentricity is calculated as a ratio.

\[ e_{ratio} = \frac{e}{d} = \frac{0.138}{2.311} = 0.060 \]

\[ e_{alw} = 0.333 \geq e_{ratio} = 0.060, \text{ SATISFACTORY} \]

- Resisting horizontal force \( H_{res} \) and its reduction by coefficient \( \gamma_S = 1.1 \):
  - \( \mu \) - reduction coefficient of contact base - soil
  - \( \mu = 1.0 \) (without reduction)

\[ F_{res} \] - resisting force
\[ F_{res} = 0 \text{ kN} \]

\[ H_{res} = \left( N \cdot \tan \varphi_d + \frac{c_d \cdot (d - 2 \cdot e)}{\mu} \right) + F_{res} = \left( 133.450 \cdot \tan(24.545) + \frac{5.714 \cdot (2.311 - 2 \cdot 0.138)}{1.0} \right) = 0 \]

\[ H_{res} = 72.571 \text{ kN/m} \]

\[ \frac{H_{res}}{\gamma_S} = \frac{72.571}{1.1} = 65.974 \text{ kN/m} \]

Result from the GEO5 – Gravity Wall program: \( H_{res} = 65.98 \text{ kNm/m} \)

- Acting horizontal force \( H_{act} \):

\[ H_{act} = T = 61.785 \text{ kN/m} \]

Result from the GEO5 – Gravity Wall program: \( H_{act} = 61.79 \text{ kNm/m} \)

- Usage:

\[ V_u = \frac{H_{act}}{H_{res}} \cdot 100 = \frac{61.785}{65.974} \cdot 100 = 93.7 \%, \text{ SATISFACTORY} \]

Result from the GEO5 – Gravity Wall program: \( V_u = 93.6 \%, \text{ SATISFACTORY} \)
Bearing Capacity of the Foundation Soil

The bearing capacity of the foundation soil is set to $R_d = 100 \, kPa$, and is compared with the stress in the inclined footing bottom.

- Usage – eccentricity:

$$V_u = \frac{e}{e_{alw}} \cdot 100 = \frac{0.060}{0.333} \cdot 100 = 18.0 \% , \text{ SATISFACTORY}$$

Result from the GEO5 – Gravity Wall program: $V_u = 18.0 \% , \text{ SATISFACTORY}$

- Stress in the footing bottom $\sigma$:

$$\sigma = \frac{N}{d - 2 \cdot e} = \frac{133.450}{2.311 - 2 \cdot 0.138} = 65.577 \, kPa$$

Result from the GEO5 – Gravity Wall program: $\sigma = 65.57 \, kPa$

- Usage:

$$V_u = \frac{\sigma}{R_d} \cdot 100 = \frac{65.577}{100} \cdot 100 = 65.6 \% , \text{ SATISFACTORY}$$

Result from the GEO5 – Gravity Wall program: $V_u = 65.6 \% , \text{ SATISFACTORY}$

Dimensioning – Wall Stem Check

In this example, a cross-section in the level of $x$-axis in Figure 5 is verified. The verified cross-section is made from plain concrete C 20/25 (characteristic cylindrical strength of concrete in compression $f_{ck} = 2000 \, kPa$, characteristic strength of concrete in tension $f_{ctm} = 2200 \, kPa$) with height $h = 1.40 \, m$ and width $b = 1.00 \, m$. The verification of a cross-section made from plain concrete is realized in accordance with EN 1992-1-1.
Calculation of the weight force and the centroid of the wall:

\[ W = 23 \cdot (0.7 \cdot 3.5 + \frac{1}{2} \cdot 0.7 \cdot 3.5) = 84.525 \text{ kN/m} \]

\[
x_i = \frac{23 \left( 0.7 \cdot 3.5 \left( \frac{0.7}{2} + 0.7 \right) + \frac{1}{2} \cdot 0.7 \cdot 3.5 \cdot \frac{2 \cdot 0.7}{3} \right)}{84.525} = 0.856 \text{ m}
\]

\[
z_i = \frac{23 \left( 0.7 \cdot 3.5 \left( -\frac{3.5}{2} \right) + \frac{1}{2} \cdot 0.7 \cdot 3.5 \left( -\frac{3.5}{3} \right) \right)}{84.525} = -1.556 \text{ m}
\]

**Calculation of the active earth pressure.** The area behind the evaluated part of the construction is divided into two sections. In the first section, the active pressure is the same as in the analysis of the whole wall. The centroids of all forces must be recalculated.

- Vertical geostatic stress \( \sigma_{z2} \) at the end of the second section:
  \[
  \sigma_{z2} = \sigma_{z1} + \gamma_2 \cdot h_2 = 27.0 + 16.561 \cdot 2.0 = 60.122 \text{ kPa}
  \]

- Active earth pressure \( \sigma_{a2b} \) at the end of the second section:
  \[
  \sigma_{a2b} = 0.4016 \cdot 60.122 - 2 \cdot 5.714 \cdot 0.5882 = 17.423 \text{ kPa}
  \]
• Resultant force of active earth pressure $S_{a2}$ and vertical and horizontal component:

(Resultant force $S_{a1}$ at the beginning is the same)

$$S_{a2} = \frac{1}{2} \cdot (17.423 - 4.121) \cdot 2.0 + 4.121 \cdot 2.0 = 21.544 \text{ kN/m}$$

$$S_{a2x} = S_{a2} \cdot \cos(\delta) = 21.544 \cdot \cos(13.636) = 20.937 \text{ kN/m}$$

$$S_{a2z} = S_{a2} \cdot \sin(\delta) = 21.544 \cdot \sin(13.636) = 5.079 \text{ kN/m}$$

• Calculation of points of action:

$x_1 = 1.400 \text{ m}$

$$z_1 = -\frac{1}{3} (1.50 - 1.38) - 2 = -2.040 \text{ m}$$

$x_2 = 1.400 \text{ m}$

$$z_2 = \frac{4.121 \cdot 2.00 \cdot \left(-\frac{2.00}{2}\right) + (17.423 - 4.121) \frac{2.00}{2} \cdot \left(-\frac{2.00}{3}\right)}{4.121 \cdot 2.00 + (17.423 - 4.121) \cdot \frac{2.00}{2}} = -0.794 \text{ m}$$

• Total resultant force of active earth pressure $S_a$ and horizontal and vertical component:

$$S_{ax} = S_{a1x} + S_{a2x} = 0.052 + 20.937 = 20.989 \text{ kN/m}$$

$$S_{az} = S_{a1z} + S_{a2z} = 0.013 + 5.079 = 5.092 \text{ kN/m}$$

$$S_a = \sqrt{S_{ax}^2 + S_{az}^2} = \sqrt{20.989^2 + 5.092^2} = 21.598 \text{ kN/m}$$

• Point of action of resultant force $S_a$:

$x_a = 1.400 \text{ m}$

$$z_a = \frac{0.052 \cdot (-2.840 + 0.800) + 20.937 \cdot (-0.794)}{0.052 + 20.937} = -0.797 \text{ m}$$

**Calculation of the water pressure.** Water pressure becomes higher with the increasing depth.

• Horizontal water pressure $\sigma_w$ in depth of 3.5 m under the surface of the adjusted terrain:

$$\sigma_w = \gamma_w \cdot (3.5 - 1.5) = 10 \cdot 2.0 = 20.000 \text{ kPa}$$
• Resultant force of water pressure $S_w$:
  \[ S_w = \frac{1}{2} \cdot \sigma_w \cdot h_w = \frac{1}{2} \cdot 20.000 \cdot 2.0 = 20.000 \text{kN/m} \]

• Point of action of resultant force $S_w$:
  \[ x_1 = 1.400 \text{ m} \]
  \[ z_1 = -\frac{1}{3} \cdot 2.0 = -0.667 \text{ m} \]

**Verification of the shear strength.** The design shear force, design normal force, design bending moment and shear strength of the cross-section are calculated. The design bending moment rotates about the middle of the verified cross-section.

• Design shear force $V_{Ed}$:
  \[ V_{Ed} = S_w + S_{ax} = 20.000 + 20.989 = 40.989 \text{kN/m} \]

  **Result from the GEO5 – Gravity Wall program:** $V_{Ed} = 40.94 \text{kN/m}$

• Design normal force $N_{Ed}$:
  \[ N_{Ed} = S_{az} + W = 5.092 + 84.525 = 89.617 \text{kN/m} \]

  **Result from the GEO5 – Gravity Wall program:** $N_{Ed} = 89.57 \text{kN/m}$

• Design bending moment $M_{Ed}$:
  \[ M_{Ed} = -W \cdot r_1 - S_{az} \cdot r_2 + S_{ax} \cdot r_3 + S_w \cdot r_4 \]
  \[ M_{Ed} = -84.525 \cdot (0.856 - 0.700) - 5.092 \cdot 0.700 + 20.989 \cdot 0.797 + 20.000 \cdot 0.667 = 13.311 \text{kNm/m} \]

  **Result from the GEO5 – Gravity Wall program:** $M_{Ed} = 13.32 \text{kNm/m}$

• Calculation of the area of compressed concrete $A_{cc}$:
  Determining the compressed area of concrete is necessary in order to determine the stresses on the front and the rear edges of the verified cross-section. The normal force $N_{Ed}$ makes compressive stress and therefore is considered to be a negative force.

  **Stress from the design normal force $N_{Ed}$**:
  \[ \frac{N}{A} = \frac{N_{Ed}}{A} = \frac{-89.617}{1.00 \cdot 1.40} = -64.012 \text{kPa} \]

  **Stress from the design bending moment $M_{Ed}$**:
\[ \pm \frac{M}{W} = \pm \frac{M_{Ed}}{W_y} = \frac{13.311}{1.00 \cdot 1.40^2} = \pm 40.748 \text{ kPa} \]

**Figure 6 Course of tension on a cross-section of a wall stem**

From Figure 6 it can be seen, that the whole area of the cross-section is compressed. 
\( A_{cc} = b \cdot h_c = 1.00 \cdot 1.40 = 1.40 \text{ m}^2 \)

- **Stress in cross-section area** \( \sigma_{cp} \):
  \[
  \sigma_{cp} = \frac{N_{Ed}}{A_{cc}} \cdot \frac{1}{1000} \cdot \frac{89.617}{1.40} \cdot \frac{1}{1000} = 0.064 \text{ MPa}
  \]

- **Design strength of concrete in compression** \( f_{cd} \):
  \[
  f_{cd} = \alpha_{cc,pl} \cdot \frac{f_{ck}}{\gamma_c} = 0.8 \cdot \frac{20.00}{1.5} = 10.667 \text{ MPa}
  \]

- **Design strength of concrete in tension** \( f_{ctd} \):
  \[
  f_{ctd} = \alpha_{ct,pl} \cdot \frac{f_{ck,005}}{\gamma_c} = \frac{0.7 \cdot f_{ctm}}{1.5} = 0.7 \cdot \frac{20.00}{1.5} = 0.821 \text{ MPa}
  \]

- **Limit stress**:
  \[
  \sigma_{c,lim} = f_{cd} - 2 \cdot \sqrt{f_{cd} \cdot (f_{cd} + f_{ctd})} = 10.667 - 2 \cdot \sqrt{0.821 \cdot (10.667 + 0.821)} = 4.525 \text{ MPa}
  \]

- **Shear strength** \( f_{cvd} \):
  \[
  f_{cvd} = \sqrt{f_{cd}^2 + \sigma_{cp} \cdot f_{cd} - \left( \frac{\max(0, \sigma_{cp} - \sigma_{c,lim})}{2} \right)^2} = \sqrt{0.821^2 + 0.064 \cdot 0.821 - \left( \frac{\max(0;0.064 - 4.525)}{2} \right)^2}
  \]

  \[ f_{cvd} = 0.852 \text{ MPa} \]
• Design shear strength $V_{rd}$:

\[ V_{rd} = \frac{f_{cd} \cdot A_{cc}}{k} \cdot 1000 = \frac{0.852 \cdot 1.4}{1.5} \cdot 1000 = 795.200 \text{ kN/m} \]

Result from the GEO5 – Gravity Wall program: $V_{rd} = 795.74 \text{ kN/m}$

• Usage:

\[ V_u = \frac{V_{Ed}}{V_{rd}} \cdot 100 = \frac{40.989}{795.200} \cdot 100 = 5.2\% \text{, SATISFACTORY} \]

Result from the GEO5 – Gravity Wall program: $V_u = 5.1\%$, SATISFACTORY

Verification of a cross-section loaded by bending moment and normal force.

• Calculation of eccentricity $e$:

\[ e = \text{Max} \left( \text{abs} \left( \frac{M_{Ed}}{N_{Ed}} \right) \cdot \frac{h}{30} : 0.02 \text{ m} \right) = \text{Max} \left( \text{abs} \left( \frac{13.311}{89.617} \right) \cdot \frac{1.400}{30} : 0.02 \right) = \text{Max}(0.149; 0.047; 0.02) \]

\[ e = 0.149 \text{ m} \]

• Effective height of cross-section $\chi$:

\[ \chi = h - 2 \cdot e = 1.400 - 2 \cdot 0.149 = 1.102 \text{ m} \]

• Design normal strength $N_{rd}$:

\[ \eta = 1.0 - \frac{\text{Max}(f_{ck} ; 50) - 50}{200} = 1.0 - \frac{\text{Max}(20; 50) - 50}{200} = 1.0 - \frac{50 - 50}{200} = 1.0 \]

\[ N_{rd} = (b \cdot \chi \cdot \eta \cdot f_{cd}) \cdot 1000 = (1.0 \cdot 1.102 \cdot 1.0 \cdot 10.667) \cdot 1000 = 11754.667 \text{ kN/m} \]

Result from the GEO5 – Gravity Wall program: $N_{rd} = 11758.60 \text{ kNm/m}$

• Usage:

\[ V_u = \frac{N_{Ed}}{N_{rd}} \cdot 100 = \frac{89.617}{11754.667} \cdot 100 = 0.8\% \text{, SATISFACTORY} \]

Result from the GEO5 – Gravity Wall program: $V_u = 0.8\%$, SATISFACTORY
2. The Second Stage – Seismic Design Situation

Verification of the Whole Wall

The second stage of calculation uses the same wall influenced by an earthquake. The calculation of earthquake effects is made according to the Mononobe-Okabe theory. The factor of horizontal acceleration is $k_h = 0.05$ (inertial force acts horizontally in an unfavourable direction) and the factor of vertical acceleration is $k_v = -0.04$ (inertial force acts downwards). The coefficients of reduction of soil parameters and the coefficients of overall stability of construction are equal to one. Therefore, the design values of soil properties are the same as the characteristic values in Table 1.

**Calculation of the weight force of the wall.** To determine the horizontal and vertical components of a force from an earthquake, it is necessary to calculate the weight force of wall without the buoyancy exerted on the wall by the groundwater. The calculation is shown in Table 4.

<table>
<thead>
<tr>
<th>Block</th>
<th>Height $h_i$ [m]</th>
<th>Width $b_i$ [m]</th>
<th>Area $A_i$ [m$^2$]</th>
<th>Block weight $\gamma_i$ [kN/m$^3$]</th>
<th>Weight force $W_i$ [kN/m]</th>
<th>Point of action $G_i \cdot x_i$</th>
<th>$G_i \cdot z_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.500</td>
<td>0.700</td>
<td>2.450</td>
<td>23</td>
<td>56.350</td>
<td>1.950</td>
<td>-2.550</td>
</tr>
<tr>
<td>2</td>
<td>3.500</td>
<td>0.700</td>
<td>1.225</td>
<td>23</td>
<td>28.175</td>
<td>1.367</td>
<td>-1.967</td>
</tr>
<tr>
<td>3</td>
<td>0.200</td>
<td>2.300</td>
<td>0.460</td>
<td>23</td>
<td>10.580</td>
<td>1.150</td>
<td>-0.700</td>
</tr>
<tr>
<td>4</td>
<td>0.600</td>
<td>2.300</td>
<td>1.380</td>
<td>23</td>
<td>31.740</td>
<td>1.150</td>
<td>-0.300</td>
</tr>
<tr>
<td>5</td>
<td>0.230</td>
<td>2.300</td>
<td>0.265</td>
<td>23</td>
<td>6.095</td>
<td>1.533</td>
<td>0.077</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>132.940</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Table 4 Dimensions, weight force and centroids of the individual blocks*

- Centroid of the structure:

\[
\begin{align*}
x_i &= \frac{\sum_{i=1}^{5} W_i \cdot x_i}{\sum_{i=1}^{5} W_i} = \frac{206.407}{132.940} = 1.553 \text{ m} \\
z_i &= \frac{\sum_{i=1}^{5} W_i \cdot z_i}{\sum_{i=1}^{5} W_i} = \frac{-215.563}{132.940} = -1.622 \text{ m}
\end{align*}
\]

- Horizontal and vertical component of the force from the earthquake:

\[
W_{eq,x} = k_h \cdot W = 0.05 \cdot 132.940 = 6.647 \text{ kN/m} \\
W_{eq,z} = -k_v \cdot W = -(0.04) \cdot 132.940 = -5.318 \text{ kN/m}
\]
Calculation of the front face resistance. The pressure at rest on the front face of the wall is the same as in the first stage of the calculation. The resultant force of the pressure at rest is $S_r = 0.430 \text{ kN/m}$.

Calculation of the active earth pressure. The course of the geostatic pressure is the same as in the first stage of calculation. In the calculation coefficients of active earth pressure $K_a$ and $K_w$ are used as characteristic values of soil properties. The active earth pressure $\sigma_a$ and the resultant force of active earth pressure $S_a$ are calculated.

- Course of geostatic stress:
  \[ \sigma_{z1} = 27.000 \text{ kPa} \]
  \[ \sigma_{z2} = 77.180 \text{ kPa} \]

- Coefficients of active earth pressure in both sections:
  ( $\alpha = 0^\circ$ - back face inclination of the structure, $\beta \neq 0^\circ$ - inclination of terrain; characteristic values of soils from Table 1 are used in calculation)

Calculation for the first layer:

\[
\beta_1 = \beta = \arctg\left(\frac{1}{10}\right) = 5.711^\circ
\]

\[
K_{a1} = \frac{\cos^2(26.5 - 0)}{\cos^2(0) \cdot \cos(0 + 15.0) \cdot \left[ 1 + \sqrt{\frac{\sin(26.5 + 15.0) \cdot \sin(26.5 - 5.711)}{\cos(0 + 15.0) \cdot \cos(0 - 5.711)}} \right]^2} = 0.3711
\]

\[
K_{ac1} = \frac{\cos(26.5) \cdot \cos(5.711) \cdot \cos(15.0 - 0) \cdot \left[ 1 + \tan(-0) \cdot \tan(5.711) \right]}{1 + \sin(26.5 + 15.0 - 0 - 5.711)} \cdot \frac{1}{\cos(15.0 + 0)} = 0.5619
\]

Calculation for the second layer:

\[
\beta_2 = \arctg\left(\frac{\gamma \cdot \tan(\beta)}{\gamma_i}\right) = \arctg\left(\frac{18.0 \cdot \tan(5.711)}{18.5}\right) = 5.557^\circ
\]

\[
K_{a2} = \frac{\cos^2(27.0 - 0)}{\cos^2(0) \cdot \cos(0 + 15.0) \cdot \left[ 1 + \sqrt{\frac{\sin(27.0 + 15.0) \cdot \sin(27.0 - 5.557)}{\cos(0 + 15.0) \cdot \cos(0 - 5.557)}} \right]^2} = 0.3631
\]

\[
K_{ac2} = \frac{\cos(27.0) \cdot \cos(5.557) \cdot \cos(15.0 - 0) \cdot \left[ 1 + \tan(-0) \cdot \tan(5.557) \right]}{1 + \sin(27.0 + 15.0 - 0 - 5.557)} \cdot \frac{1}{\cos(15.0 + 0)} = 0.5563
\]
• Calculation of height in the first layer of soil MS, where the active earth pressure is neutral:

\[
h_0 = \frac{2 \cdot c_{ef,1} \cdot K_{ac,1}}{\gamma_1 \cdot K_{at}} = \frac{2 \cdot 12 \cdot 0.5619}{18.0 \cdot 0.3711} = 2.019 \text{ m} > 1.50 \text{ m}
\]

• Active earth pressure \( \sigma_a \) is calculated only for the second section (in the first section it’s equal to zero):

\[
\sigma_{a2a} = \sigma_{z1} \cdot K_{a2} - 2 \cdot c_{ef,at} \cdot K_{ac,2} = 27.00 \cdot 0.3631 - 2 \cdot 8.0 \cdot 0.5563 = 0.903 kPa
\]

\[
\sigma_{a2b} = \sigma_{z2} \cdot K_{a2} - 2 \cdot c_{ef,at} \cdot K_{ac,2} = 77.18 \cdot 0.3631 - 2 \cdot 8.0 \cdot 0.5563 = 19.123 kPa
\]

• Resultant force of the active earth pressure \( S_a \) and its horizontal and vertical components:

\[
S_a = \frac{1}{2} \cdot (\sigma_{a2b} - \sigma_{a2a}) \cdot h_2 + \sigma_{a2a} \cdot h_2 = \frac{1}{2} \cdot (19.123 - 0.903) \cdot 3.03 + 0.903 \cdot 3.03 = 30.339 kN/m
\]

\[
S_{ax} = S_a \cdot \cos(\delta) = 30.339 \cdot \cos(15.0) = 29.306 kN/m
\]

\[
S_{az} = S_a \cdot \sin(\delta) = 30.339 \cdot \sin(15.0) = 7.852 kN/m
\]

• Point of action of the resultant force:

\[
x = 2.300 \text{ m}
\]

\[
z = \frac{0.903 \cdot 3.03 \cdot \left(-\frac{3.03}{2} + 0.23\right) + (19.123 - 0.903) \cdot \frac{3.03}{2} \cdot \left(-\frac{3.03}{3} + 0.23\right)}{0.903 \cdot 3.03 + (19.123 - 0.903) \cdot \frac{3.03}{2}} = -0.826 \text{ m}
\]

**Increase of the active earth pressure caused by an earthquake.** An earthquake increases the effect of active earth pressure.

• Calculation of the seismic inertia angle in the first layer (without restricted water influence):

\[
\psi_1 = \arctg \left( \frac{k_h}{1 - k_v} \right) = \arctg \left( \frac{0.05}{1 - (-0.04)} \right) = 2.752^\circ
\]

• Calculation of the seismic inertia angle in the second layer (with restricted water influence):

\[
\psi_2 = \arctg \left( \frac{\gamma_{sat,2} \cdot k_h}{\gamma_{sat,2} \cdot (1 - k_v)} \right) = \arctg \left( \frac{20.5 \cdot 0.05}{20.5 - 10 \cdot [1 - (-0.04)]} \right) = 5.362^\circ
\]
• Coefficient $K_{ae}$ for the active earth pressure in both sections:

$$K_{ae} = \frac{\cos^2(\varphi - \psi - \alpha)}{\cos \psi \cdot \cos^2(\alpha) \cdot \cos(\psi + \alpha + \delta) \cdot \left(1 + \frac{\sin(\varphi + \delta) \cdot \sin(\varphi - \psi + \beta)}{\cos(\alpha + \delta + \psi) \cdot \cos(\alpha - \beta)}\right)^2}$$

$$K_{ae1} = 0.4102$$

$$K_{ae2} = \frac{\cos^2(26.5 - 2.752 - 0)}{\cos(2.752) \cdot \cos^2(0) \cdot \cos(2.752 + 0 + 15.0) \cdot \left(1 + \frac{\sin(26.5 + 15.0) \cdot \sin(26.5 - 2.752 - 5.711)}{\cos(0 + 15.0 + 2.752) \cdot \cos(0 - 5.711)}\right)^2}$$

• Calculation of normal stress $\sigma_d$ from the earthquake effects. The normal stress is calculated from the bottom of the wall:

$\sigma_{d1} = 0.000 \text{ kPa} \cdot \text{normal stress in the footing bottom}$

$$\sigma_{d1} = \gamma \cdot h_2 \cdot (1 - k_r) = 16.561 \cdot 3.03 \cdot [1 - (-0.04)] = 52.187 \text{ kPa}$$

$$\sigma_{d0} = \sigma_{d1} + \gamma_1 \cdot h_1 \cdot (1 - k_r) = 52.187 + 18.0 \cdot 1.50 \cdot [1 - (-0.04)] = 80.267 \text{ kPa}$$

• Increase of the active earth pressure caused by the earthquake in both sections:

$$\sigma_{ae1a} = \sigma_{d0} \cdot (K_{ae1} - K_{a1}) = 80.267 \cdot (0.4102 - 0.3711) = 3.138 \text{ kPa}$$

$$\sigma_{ae1b} = \sigma_{d1} \cdot (K_{ae1} - K_{a1}) = 52.187 \cdot (0.4102 - 0.3711) = 2.041 \text{ kPa}$$

$$\sigma_{ae2a} = \sigma_{d1} \cdot (K_{ae2} - K_{a2}) = 52.187 \cdot (0.4429 - 0.3631) = 4.165 \text{ kPa}$$

• Resultant forces of the increase of the active earth pressure $S_{ae}$ in both sections:

$$S_{ae1} = \frac{1}{2} \cdot (\sigma_{ae1a} - \sigma_{ae1b}) \cdot h_1 + \sigma_{ae1b} \cdot h_1 = \frac{1}{2} \cdot (3.138 - 2.041) \cdot 1.50 + 2.041 \cdot 1.50 = 3.884 \text{ kN/m}$$

$$S_{ae1x} = S_{ae1} \cdot \cos(\delta) = 3.884 \cdot \cos(15.0) = 3.752 \text{ kN/m}$$

$$S_{ae1z} = S_{ae1} \cdot \sin(\delta) = 3.884 \cdot \sin(15.0) = 1.005 \text{ kN/m}$$

$$S_{ae2} = \frac{1}{2} \cdot \sigma_{ae2a} \cdot h_2 = \frac{1}{2} \cdot 4.165 \cdot 3.03 = 6.310 \text{ kN/m}$$
\[ S_{ae,2x} = S_{ae2} \cdot \cos(\delta) = 6.310 \cdot \cos(15.0) = 6.095 \, kN / m \]

\[ S_{ae,2z} = S_{ae2} \cdot \sin(\delta) = 6.310 \cdot \sin(15.0) = 1.633 \, kN / m \]

- Points of action of the resultant forces:
  \[ x_1 = 2.300 \, m \]
  \[ z_1 = \frac{2.041 \cdot 1.50 \cdot \left( - \frac{1.50}{2} - (3.03 - 0.23) \right) + (3.138 - 2.041) \frac{1.50}{2} \cdot \left( \frac{2}{3} \cdot (-1.50) - (3.03 - 0.23) \right)}{2.041 \cdot 1.50 + (3.138 - 2.041) \cdot \frac{1.50}{2}} \]

  \[ z_1 = -3.603 \, m \]

  \[ x_2 = 2.300 \, m \]

  \[ z_2 = \frac{2}{3} \cdot (-3.03) - 0.23 = -1.790 \, m \]

- Total resultant force of the increase of active earth pressure \( S_{ae} \) and its horizontal and vertical component:
  \[ S_{ae,x} = S_{ae1x} + S_{ae2x} = 3.752 + 6.095 = 9.847 \, kN / m \]

  \[ S_{ae,z} = S_{ae1z} + S_{ae2z} = 1.005 + 1.633 = 2.638 \, kN / m \]

  \[ S_{ae} = \sqrt{S_{ae,x}^2 + S_{ae,z}^2} = \sqrt{9.847^2 + 2.638^2} = 10.194 \, kN / m \]

- Point of action of the resultant force \( S_{ae} \):
  \[ x_{ae} = 2.300 \, m \]

  \[ z_{ae} = \frac{3.752 \cdot (-3.603) + 6.095 \cdot (-1.790)}{3.752 + 6.095} = -2.481 \, m \]

**Calculation of water pressure.** The water pressure is the same as in the verification of the whole wall in the first stage. The resultant force of the water pressure is \( S_w = 33.330 \, kN / m \) and has the same point of action as in the first stage.

**Calculation of the hydrodynamic pressure acting on the front face of the wall.** The action of the hydrodynamic pressure caused by the earthquake is calculated from the groundwater table to the bottom of the wall. The direction of the force is the same as the direction of the horizontal acceleration.
Calculation of the resultant force of the hydrodynamic pressure \( P_{wd} \) caused by the earthquake:

\[
H = 0.6 + 0.23 = 0.83 \text{ m}
\]

\[
P_{wd} = \frac{7}{12} \cdot k_p \cdot y_w \cdot H^2 = \frac{7}{12} \cdot 0.05 \cdot 10.0 \cdot 0.83^2 = 0.201 \text{ kN/m}
\]

Point of action of the resultant force \( P_{wd} \):

\[
x = 2.300 \text{ m}
\]

\[
z = y_{wd} - 0.23 = (0.4 \cdot H) - 0.23 = (0.4 \cdot 0.83) - 0.23 = 0.102 \text{ m}
\]

Checking for overturning stability. The moments calculated in the analysis rotate about the origin of the coordinate system (left bottom corner of the structure). Resisting moment \( M_{res} \) and overturning moment \( M_{ovr} \) are calculated for verification.

Calculation of the resisting moment \( M_{res} \):

\[
M_{res} = W \cdot r_1 + W_{eq,z} \cdot r_2 + S_{az} \cdot r_3 + S_{wc,z} \cdot r_4 = 116.484 \cdot 1.601 + 5.318 \cdot 1.553 + 7.852 \cdot 2.300 + 2.638 \cdot 2.300
\]

\[
M_{res} = 116.484 \cdot 1.601 + 5.318 \cdot 1.553 + 7.852 \cdot 2.300 + 2.638 \cdot 2.300
\]

\[
M_{res} = 218.877 \text{ kNm/m}
\]

Result from the GEO5 – Gravity Wall program: \( M_{res} = 218.86 \text{ kNm/m} \)

Calculation of the overturning moment \( M_{ovr} \):

\[
M_{ovr} = -S_r \cdot r_1 + W_{eq,z} \cdot r_2 + S_{az} \cdot r_3 + S_{wc,z} \cdot r_4 + S_w \cdot r_5 + P_{wd} \cdot r_6
\]

\[
M_{ovr} = -0.43 \cdot 0.200 + 6.647 \cdot 1.622 + 29.306 \cdot 0.826 + 9.847 \cdot 2.481 + 33.330 \cdot 1.056 + 0.201 \cdot 0.102
\]

\[
M_{ovr} = 94.550 \text{ kNm/m}
\]

Result from the GEO5 – Gravity Wall program: \( M_{ovr} = 94.59 \text{ kNm/m} \)
Usage:
\[ V_u = \frac{M_{\text{u}}}{M_{\text{res}}} \times 100 = \frac{94.550}{218.877} \times 100 = 43.2\% \text{, SATISFACTORY} \]

Result from the GEO5 – Gravity Wall program: \( V_u = 43.2\% \text{, SATISFACTORY} \)

**Checking for slip.** The slip in the inclined footing bottom in 1:10 inclination is checked (Figure 4.).

Total vertical and horizontal forces \( F_{\text{ver}} \) a \( F_{\text{hor}} \):
\[ F_{\text{ver}} = 116.484 + 5.318 + 7.852 + 2.638 = 132.292 \text{ kN/m} \]
\[ F_{\text{hor}} = -0.43 + 6.647 + 29.606 + 9.847 + 33.33 = 79.000 \text{ kN/m} \]

Normal force in the footing bottom \( N \):
\[ \alpha_b = 5.711^\circ \]
\[ N = F_{\text{ver}} \cdot \cos(\alpha_b) + F_{\text{hor}} \cdot \sin(\alpha_b) = 132.292 \cdot \cos(5.711) + 79.000 \cdot \sin(5.711) = 139.496 \text{ kN/m} \]

Shear force in the footing bottom \( T \):
\[ T = -F_{\text{ver}} \cdot \sin(\alpha_b) + F_{\text{hor}} \cdot \cos(\alpha_b) = -132.292 \cdot \sin(5.711) + 79.000 \cdot \cos(5.711) = 65.444 \text{ kN/m} \]

Eccentricity of the normal force:
\[ d - \text{inclined width of the footing bottom} \]
\[ e_{\text{alw}} - \text{maximal allowable eccentricity} \]
\[ d = \frac{2.3}{\cos(\alpha_b)} = \frac{2.3}{\cos(5.711)} = 2.311 \text{ m} \]
\[ e = \frac{M_{\text{u}} - M_{\text{res}} + \frac{N \cdot d}{2}}{N} = \frac{94.550 - 218.877 + \frac{139.496 \cdot 2.311}{2}}{139.496} = 0.264 \text{ m} \]

In the program, the eccentricity is calculated as a ratio.
\[ e_{\text{ratio}} = \frac{e}{d} = \frac{0.264}{2.311} = 0.114 \]
\[ e_{\text{alw}} = 0.333 \geq e_{\text{ratio}} = 0.114 \text{ , SATISFACTORY} \]

Resisting horizontal force \( H_{\text{res}} \) and its reduction by coefficient \( \gamma_\delta = 1.1 \):
\[ \mu - \text{reduction coefficient of contact base - soil} \]
\[ \mu = 1.0 \text{ (without reduction)} \]
\[ F_{res} \text{ - resisting force} \]
\[ F_{res} = 0 \text{ kN} \]

\[ H_{res} = \left( N \cdot \tan \varphi_{ef} \cdot \frac{c_{ef}}{\mu} \right) + F_{res} = \left( 139.496 \cdot \tan(27.0) + \frac{8.00 \cdot (2.311 - 2 \cdot 0.264)}{1.0} \right) + 0 \]

\[ H_{res} = 85.341 \text{ kN/m} \]

**Result from the GEO5 – Gravity Wall program:** \( H_{res} = 85.33 \text{ kNm/m} \)

- **Acting horizontal force** \( H_{act} \):

\[ H_{act} = T = 65.444 \text{ kN/m} \]

**Result from the GEO5 – Gravity Wall program:** \( H_{act} = 65.37 \text{ kNm/m} \)

- **Usage:**

\[ V_u = \frac{H_{act}}{H_{res}} \cdot 100 = \frac{65.444}{85.341} \cdot 100 = 76.7 \%, \text{ SATISFACTORY} \]

**Result from the GEO5 – Gravity Wall program:** \( V_u = 76.6 \%, \text{ SATISFACTORY} \)

**Bearing Capacity of the Foundation Soil**

The bearing capacity of the foundation soil is set to \( R_d = 100 \text{ kPa} \), and is compared with the stress in the inclined footing bottom.

- **Usage – eccentricity:**

\[ V_u = \frac{e}{e_{alw}} \cdot 100 = \frac{0.114}{0.333} \cdot 100 = 34.2 \%, \text{ VYHOVUJE} \]

**Result from the GEO5 – Gravity Wall program:** \( V_u = 34.6 \%, \text{ SATISFACTORY} \)

- **Stress in the footing bottom** \( \sigma \):

\[ \sigma = \frac{N}{d - 2 \cdot e} = \frac{139.496}{2.311 - 2 \cdot 0.264} = 78.237 \text{ kPa} \]

**Result from the GEO5 – Gravity Wall program:** \( \sigma = 78.29 \text{ kPa} \)
• Usage:

\[ V_u = \frac{\sigma}{R_d} \cdot 100 = \frac{78.237}{100} \cdot 100 = 78.2\text{\%} \text{, SATISFACTORY} \]

Result from the GEO5 – Gravity Wall program: \( V_u = 78.3\text{\%} \text{, SATISFACTORY} \)

Dimensioning – Wall Stem Check

In this example, the cross-section on the level of the \( x \)-axis in Figure 5 is verified. The cross-section is made from plain concrete C 20/25 (characteristic cylindrical strength of concrete in compression \( f_{ck} = 20000\text{ kPA} \), characteristic strength of concrete in tension \( f_{ctm} = 2200\text{ kPA} \)) with height \( h = 1.40\text{ m} \) and width \( b = 1.00\text{ m} \). The verification of the cross-section made from plain concrete is realized in accordance with EN 1992-1-1.

• Calculation of the weight force and the centroid is the same as in the first stage:

\[ W = 84.525\text{ kN/m} \]

\[ x_c = 0.856\text{ m} \]

\[ z_c = -1.556\text{ m} \]

• Horizontal and vertical component of the force caused by the earthquake (the centroid is the same as the centroid of the weight force):

\[ W_{eq,x} = k_h \cdot W = 0.05 \cdot 84.525 = 4.226\text{ kN/m} \]

\[ W_{eq,z} = -k_v \cdot W = (-0.04) \cdot 84.525 = -3.381\text{ kN/m} \]

Calculation of the active earth pressure. The area behind the evaluated part of the construction is divided into two sections. The centroids of all forces must be recalculated.

• Vertical geostatic stress \( \sigma_{z1} \) and \( \sigma_{z2} \) in both sections:

\[ \sigma_{z1} = \gamma_1 \cdot h_2 = 18.0 \cdot 1.5 = 27.000\text{ kPa} \]

\[ \sigma_{z2} = \sigma_{z1} + \gamma_2 \cdot h_2 = 27.0 + 16.561 \cdot 2.0 = 60.122\text{ kPa} \]

• Active earth pressure in the second section \( \sigma_{a2a} \) and \( \sigma_{a2b} \) (the active earth pressure in the first section is equal to zero):

\[ \sigma_{a2a} = 0.3631 \cdot 27.000 - 2 \cdot 8.0 \cdot 0.5563 = 0.903\text{ kPa} \]

\[ \sigma_{a2b} = 0.3631 \cdot 60.122 - 2 \cdot 8.0 \cdot 0.5563 = 12.929\text{ kPa} \]
Total resultant force of the active earth pressure $S_a$ and its horizontal and vertical components:

$S_a = \frac{1}{2} \cdot (12.929 - 0.903) \cdot 2.0 + 0.903 \cdot 2.0 = 13.832 \text{ kN/m}$

$S_{a, x} = S_{a, 2} \cdot \cos(\delta) = 13.832 \cdot \cos(15.0) = 13.360 \text{ kN/m}$

$S_{a, z} = S_{a, 2} \cdot \sin(\delta) = 13.832 \cdot \sin(15.0) = 3.580 \text{ kN/m}$

Point of action of the resultant force $S_a$

$x = 1.400\text{ m}$

$z = \frac{0.903 \cdot 2.00 \cdot \left( -\frac{2.00}{2} \right) + (12.929 - 0.903) \cdot \frac{2.00}{2} \cdot \left( -\frac{2.00}{3} \right)}{0.903 \cdot 2.00 + (12.929 - 0.903) \cdot \frac{2.00}{2}} = -0.710\text{ m}$

**Increase of the active earth pressure caused by an earthquake.** An earthquake increases the effect of the active earth pressure.

Calculation of the normal stress $\sigma_d$ from the earthquake effects. The vertical pressure is calculated from the lower part of the stem:

$\sigma_{d, 2} = 0.000 \text{ kPa} \cdot \text{ normal stress at the level of the lower part of the stem}$

$\sigma_{d, 1} = \gamma_2 \cdot (h_2) \cdot (1 - k_v) = 16.561 \cdot (2.00) \cdot \left[ 1 - (-0.04) \right] = 34.447 \text{ kPa}$

$\sigma_{d, 0} = \sigma_{d, 1} + \gamma_1 \cdot h_1 \cdot (1 - k_v) = 34.447 + 18.0 \cdot 1.50 \cdot \left[ 1 - (-0.04) \right] = 62.527 \text{ kPa}$

Increase of the active earth pressure caused by the earthquake effects in both sections:

$\sigma_{ae, 1a} = \sigma_{d, 0} \cdot (K_{ae1} - K_{at}) = 62.527 \cdot (0.4102 - 0.3711) = 2.445 \text{ kPa}$

$\sigma_{ae, 1b} = \sigma_{d, 1} \cdot (K_{ae1} - K_{at}) = 34.447 \cdot (0.4102 - 0.3711) = 1.347 \text{ kPa}$

$\sigma_{ae, 2a} = \sigma_{d, 1} \cdot (K_{ae2} - K_{at}) = 34.447 \cdot (0.4429 - 0.3631) = 2.749 \text{ kPa}$

Resultant forces of the increase of the active earth pressure $S_{ae}$ in both sections:

$S_{ae} = \frac{1}{2} \cdot (\sigma_{ae, 1a} - \sigma_{ae, 1b}) \cdot h_1 + \sigma_{ae, 1b} \cdot h_1 = \frac{1}{2} \cdot (2.445 - 1.347) \cdot 1.50 + 1.347 \cdot 1.50 = 2.844 \text{ kN/m}$

$S_{ae, 1a} = \sigma_{ae} \cdot \cos(\delta) = 2.844 \cdot \cos(15.0) = 2.747 \text{ kN/m}$
\[ S_{aer1} = S_{aer} \cdot \sin(\delta) = 2.844 \cdot \sin(15.0) = 0.736 \, kN/m \]

\[ S_{aer2} = \frac{1}{2} \cdot \sigma_{aer,2a} \cdot h_2 = \frac{1}{2} \cdot 2.749 \cdot 2.00 = 2.749 \, kN/m \]

\[ S_{aer2x} = S_{aer2} \cdot \cos(\delta) = 2.749 \cdot \cos(15.0) = 2.655 \, kN/m \]

\[ S_{aer2z} = S_{aer2} \cdot \sin(\delta) = 2.749 \cdot \sin(15.0) = 0.711 \, kN/m \]

- Points of action of the resultant forces:
  \[ x_1 = 1.400 \, m \]

\[ z_1 = \frac{1.347 \cdot 1.50 \cdot \left( -\frac{1.50}{2} - 200 \right) + (2.445 - 1.347) \cdot \frac{1.50}{2} \cdot \left( \frac{2}{3} \cdot (-1.50) - 2.00 \right)}{1.347 \cdot 1.50 + (2.445 - 1.347) \cdot \frac{1.50}{2}} = -2.824 \, m \]

\[ x_2 = 1.400 \, m \]

\[ z_2 = \frac{2}{3} \cdot (-2.0) = -1.333 \, m \]

- Total resultant force \( S_{ae} \) and its horizontal and vertical components:
  \[ S_{ae,x} = S_{ae1x} + S_{ae2x} = 2.747 + 2.655 = 5.402 \, kN/m \]

\[ S_{ae,z} = S_{ae1z} + S_{ae2z} = 0.736 + 0.711 = 1.447 \, kN/m \]

\[ S_{ae} = \sqrt{S_{ae,x}^2 + S_{ae,z}^2} = \sqrt{5.402^2 + 1.447^2} = 5.592 \, kN/m \]

- Point of action of the resultant force \( S_{ae} \):
  \[ x_{ae} = 1.400 \, m \]

\[ z_{ae} = \frac{2.747 \cdot (-2.824) + 2.655 \cdot (-1.333)}{2.747 + 2.655} = -2.091 \, m \]

**Calculation of the water pressure.** Water pressure becomes higher with the increasing depth.

- Horizontal water pressure \( \sigma_w \) in depth of 3.5 m under the surface of the adjusted terrain:
  \[ \sigma_w = \gamma_w \cdot (3.5 - 1.5) = 10 \cdot 2.0 = 20.000 \, kPa \]

- Resultant force of the water pressure \( S_w \):
\[ S_w = \frac{1}{2} \cdot \sigma_w \cdot h_w = \frac{1}{2} \cdot 20.000 \cdot 2.0 = 20.000 \text{kN/m} \]

- **Point of action of the resultant force** \( S_w \):
  \[ x_1 = 1.400 \text{ m} \]
  \[ z_1 = -\frac{1}{3} \cdot 2.0 = -0.667 \text{ m} \]

**Verification of shear strength.** The design shear force, design normal force, design bending moment and shear strength of the cross-section are calculated. The design bending moment rotates about the middle of the verified cross-section.

- **Design shear force** \( V_{Ed} \):
  \[ V_{Ed} = S_w + S_{a,c} + S_{ae,c} + W_{eq,z} = 20.000 + 13.360 + 5.402 + 4.226 = 42.988 \text{kN/m} \]

Result from the GEO5 – Gravity Wall program: \( V_{Ed} = 42.95 \text{kN/m} \)

- **Design normal force** \( N_{Ed} \):
  \[ N_{Ed} = S_{a,c} + S_{ae,c} + W + W_{eq,z} = 3.580 + 1.447 + 84.525 + 3.381 = 92.933 \text{kN/m} \]

Result from the GEO5 – Gravity Wall program: \( N_{Ed} = 92.89 \text{kN/m} \)

- **Design bending moment** \( M_{Ed} \):
  \[ M_{Ed} = -W \cdot r_1 - S_{a,c} \cdot r_2 + S_{a,c} \cdot r_3 + S_w \cdot r_4 - W_{eq,z} \cdot r_5 - S_{ae,c} \cdot r_6 + W_{eq,c} \cdot r_7 + S_{ae,c} \cdot r_8 \]
  \[ M_{Ed} = -84.525 \cdot (0.856 - 0.700) - 3.580 \cdot 0.700 + 13.360 \cdot 0.710 + 2.0 \cdot 0.667 - 3.381 \cdot (0.856 - 0.700) - 1.447 \cdot 0.700 + 4.226 \cdot 1.556 + 5.402 \cdot 2.091 \]
  \[ M_{Ed} = 23.458 \text{kNm/m} \]

Result from the GEO5 – Gravity Wall program: \( M_{Ed} = 23.46 \text{kNm/m} \)

- **Calculation of the area of compressed concrete** \( A_{cc} \):
  Determining the compressed area of concrete is necessary in order to determine the stresses on the front and the rear edges of the verified cross-section. The normal force \( N_{Ed} \) makes compressive stress and therefore is considered to be a negative force.

  Stress from design normal force \( N_{Ed} \):
  \[ \frac{N}{A} = \frac{N_{Ed}}{A} = -\frac{92.933}{1.00 \cdot 1.40} = -66.381 \text{kPa} \]
Stress from design bending moment $M_{Ed}$:

$$\pm \frac{M}{W} = \pm \frac{M_{Ed}}{W_y} = \frac{\frac{23.458}{1.00 \cdot 1.40^2}}{\frac{1}{6}} = \pm 71.810 \text{ kPa}$$

**Figure 7** Course of tension on the cross-section of wall stem

From Figure 7 it can be seen, that not the whole cross section is compressed, only a part, $h_c$:

$$h_c = \frac{138.191}{138.191 + 5.429} = 1.347 \text{ m}$$

$$A_{cc} = b \cdot h_c = 1.00 \cdot 1.347 = 1.347 \text{ m}^2$$

- Stress on the cross-section area $\sigma_{cp}$:

$$\sigma_{cp} = \frac{N_{Ed}}{A_{cc}} \cdot \frac{1}{1000} = 92.933 \cdot \frac{1}{1000} = 0.06899 \text{ MPa}$$

- Design strength of concrete in compression $f_{cd}$:

$$f_{cd} = \alpha_{cc,pl} \cdot \frac{f_{ck}}{\gamma_c} = 0.8 \cdot \frac{20.00}{1.5} = 10.667 \text{ MPa}$$

- Design strength of concrete in tension $f_{ct}$:

  $f_{ct,k,005}$ - lower value of characteristic strength of concrete in tension

$$f_{ctd} = \alpha_{ct,pl} \cdot \frac{f_{ct,k,005}}{\gamma_c} = \alpha_{ct,pl} \cdot \frac{0.7 \cdot f_{ctm}}{\gamma_c} = 0.8 \cdot \frac{0.7 \cdot 2.20}{1.5} = 0.821 \text{ MPa}$$
• Limit stress:
\[ \sigma_{\text{e,lim}} = f_{cd} - 2 \cdot \sqrt{f_{cd} \cdot (f_{cd} + f_{cbd})} = 10.667 - 2 \cdot \sqrt{0.821 \cdot (10.667 + 0.821)} = 4.525 \, \text{MPa} \]

• Shear strength \( f_{cbd} \):
\[ f_{cbd} = \sqrt{\frac{f_{cbd}^2 + \sigma_{cp} \cdot f_{cbd} - \left( \frac{\max(0, \sigma_{cp} - \sigma_{e,lim})}{2} \right)^2}{0.821^2 + 0.06899 \cdot 0.821 - \left( \frac{\max(0; 0.066 - 4.525)}{2} \right)^2}} \]
\[ f_{cbd} = 0.855 \, \text{MPa} \]

• Design shear strength \( V_{rd} \):
\[ k = 1.5 \]
\[ V_{rd} = \frac{f_{cbd} \cdot A_{cc}}{k} \cdot 1000 = \frac{0.855 \cdot 1.347}{1.5} \cdot 1000 = 767.790 \, \text{kN/m} \]

Result from the GEO5 – Gravity Wall program: \( V_{rd} = 767.58 \, \text{kN/m} \)

• Usage:
\[ V_u = \frac{V_{Ed}}{V_{rd}} \cdot 100 = \frac{42.988}{767.790} \cdot 100 = 5.6 \% , \text{ SATISFACTORY} \]

Result from the GEO5 – Gravity Wall program: \( V_u = 5.6 \% , \text{ SATISFACTORY} \)

Verification of a cross-section loaded by bending moment and normal force.

• Calculation of eccentricity \( e \):
\[ e = \text{Max} \left( \frac{\text{abs} \left( \frac{M_{Ed}}{N_{Ed}} \right) \cdot h}{30} ; 0.02 \, \text{m} \right) = \text{Max} \left( \frac{\text{abs} \left( \frac{23.458}{92.933} \right) \cdot 1.400}{30} ; 0.02 \right) = \text{Max}(0.252; 0.047; 0.02) \]
\[ e = 0.252 \, \text{m} \]

• Effective high of cross-section \( \chi \):
\[ \chi = h - 2 \cdot e = 1.400 - 2 \cdot 0.252 = 0.896 \, \text{m} \]

• Design normal strength \( N_{rd} \):
\[ \eta = 1.0 - \frac{\text{Max}(f_{ck}; 50) - 50}{200} = 1.0 - \frac{\text{Max}(20; 50) - 50}{200} = 1.0 - \frac{50 - 50}{200} = 1.0 \]
\[ N_{rd} = (b \cdot \chi \cdot \eta \cdot f_{cd}) \cdot 1000 = (1.0 \cdot 0.896 \cdot 1.0 \cdot 10.667) \cdot 1000 = 9557.632 \, \text{kN/m} \]

Result from the GEO5 – Gravity Wall program: \( N_{rd} = 9543.22 \, \text{kNm/m} \)

• Usage:
\[ V_u = \frac{N_{Ed}}{N_{Ref}} \cdot 100 = \frac{92.933}{9557.632} \cdot 100 = 1.0 \% \], SATISFACTORY

Result from the GEO5 – Gravity Wall program: \[ V_u = 1.0 \% \], SATISFACTORY